March, 1983

LIDS-TH-1288

Research Supported By:
Office of Naval Research
Contract ONR/N00014-77-C-0532
(NR 041-519)

OPTIMAL SHIP POSITIONS FOR NAVAL BATTLE GROUP DEFENSE PROBLEMS

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Optimal Ship Positions For Naval Battle Group Defense Problems		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER LIDS-TH-1288
7. AUTHOR(s) Robin Cynthia Magonet-Neray		8. CONTRACT OR GRANT NUMBER(s) ONR/N00014-77-C-0532
9. PERFORMING ORGANIZATION NAME AND ADDRESS M.I.T. Laboratory for Information and Decision Systems Cambridge, MA 02139		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Navy Office of Naval Research Arlington, VA 22217		12. REPORT DATE March 83
		13. NUMBER OF PAGES 204
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Unlimited

This document has been approved for public release and sale; its distribution is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Approved for public release; distribution unlimited.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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This report is based on the unaltered thesis of Robin Cynthia Magonet-Neray, submitted in partial fulfillment of the requirements for the degree of Master of Science in Operations Research at the Massachusetts Institute of Technology in September 1983. The research was conducted at the M.I.T. Laboratory for Information and Decision Systems, with support provided in part by the Office of Naval Research under the contract ONR/N00014-77-C-0532 (NR 041-519).

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B.Sc. McGill University

(1978)

Submitted to the Department of
Electrical Engineering
in Partial Fulfillment of the
Requirements of the
Degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 1983

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Department of Electrical Engineering
March 16, 1983

Certified by Michael Athans
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Accepted by Richard C. Larson
Co-Director, Operations Research Center

ABSTRACT

OPTIMAL SHIP POSITIONS FOR NAVAL BATTLE GROUP DEFENSE PROBLEMS

by

Robin Cynthia Magonet-Neray

Submitted to the Department of Electrical Engineering on March 16, 1983 in partial fulfillment of the requirements for the Degree of Master of Science in Operations Research

This report presents an optimization model to maximize the survival probability of a carrier operating in a Battle Group (BG) environment given AAW (anti-air warfare) and ASW (anti-sub warfare) resources. The model is a static, probabilistic, two-dimensional representation.

~~We develop~~ several probabilistic and parametric relationships, including:
[illegible]

(1) the probability that a ship kills a target
(2) the optimum intercept distance
(3) the effect of the ships' distance to the carrier
(4) the probability distributions of the targets
And (5) the survival probability of the carrier.

The solution to the problem is the optimum location of the AAW and ASW ships with respect to the carrier; those locations that maximize the probability of survival of the carrier, from the AAW and ASW threats.

We perform numerical experiments when both one and many ships are defending the carrier. We study the quantitative impact on the survival probability of the carrier and on the optimum location of the ship(s) when we do parametric analyses that reflect changes in

(a) threat sector locations
(b) target density in each sector
(c) AAW and ASW defense capabilities on a per ship basis
(d) number of defending ships

The model, although extraordinarily simple, performs in a very reasonable way.

Thesis Supervisor: Dr. Michael Athans
Title: Professor of Electrical Engineering

ACKNOWLEDGEMENTS

I am deeply grateful to Professor Michael Athans for his insight and direction, for his constant encouragement, and for our many productive discussions.

I also want to thank my mother and father, my sister, Shelley, my brother Gordon, and my brother-in-law, Issy for their unbounded support. Georges, Ruth and Michel Neray (my extended family!) were kind enough to inject the amount of humour required when undertaking a project of this size.

I would also like to acknowledge my dear friends, Glenda Consla, Marjorie Lyon, David Martin, Norio Kushi, Ruth Bardenstein, Shannon Peet, Carrie Schipper and Shary Stamm, all of whom provided unlimited patience and support. I thank Bobby, Joe and Bobby at User Accounts for their general enthusiasm, comraderie and understanding.

Finally, I want to thank my devoted husband, Phil, for reading the manuscript and offering helpful suggestions. He was a constant source of inspiration.

This research was performed at the M.I.T. Laboratory for Information and Decision Systems with support provided by the Office of Naval research under the contract ONR/N00014 - 77 - C - 0532 (NR 041 - 519).

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CHAPTER 1 INTRODUCTION

1.1 MOTIVATION AND PURPOSE

With the rising cost and sophistication of aircraft carriers, it is increasingly more important to find better ways to protect them.

Typically, a carrier is defended against air and submarine threats by ships with simultaneous AAW (anti-air warfare), ASW (anti-submarine warfare) and ASUW (anti-surface warfare) capabilities. These ships are located in some "neighbourhood" of the carrier. With the aid of advanced radar and sonar, these ships are required to detect and kill enemy air, surface and sub targets. Clearly, the locations of these ships relative to the carrier determines their effectiveness against enemy threats.

The purpose of this report is to develop a preliminary mathematical model to study such problems. For the sake of simplicity, we focus attention only on the AAW and ASW threats. The model is a static, probabilistic and two-dimensional representation. Its goal is to maximize the probability of survival of the carrier given air and sub threats, by finding the optimum locations of the defending ships, relative to the carrier. We test the performance of the model through comprehensive numerical studies.

We wish to stress at the outset that the mathematical model as presented here cannot be used for realistic ship location decision-aids for an actual battle group. The models that we have used are too crude to properly capture the AAW and ASW surveillance and weapons effectiveness of actual platforms (destroyers, frigates, cruisers, etc.). Also, the numerical values that have been used in the models, do not represent any real platform parameters (which are classified). The real intent of this research was to understand the complexities of the optimization problem involved if a more realistic model were to be used.

The Battle Group location problem is a very important problem which is amenable to quantitative analysis and optimization, Athans (1), (2). However, this problem has not been analyzed in the unclassified literature, except for the very recent study reported by Castanon et al (3), which is in the same spirit as the research reported herein. However, the quantitative defense effectiveness models in (3) are quite different.

In spite of the limitations of these models, they offer valuable guidance on how to exploit the multiwarfare capabilities of modern naval platforms and to use them in a coordinated fashion in the defense of a modern Battle Group. We quote below a paragraph of the paper (4) by R.F. Schoultz (RADM, USN) that provides ample justification for the research.

" The situation for surface combatants is perhaps the most illustrative. With the shrinking of the force levels there is a need to share platforms among the missions of AAW, ASW, and ASUW. Depending upon the scenario, these forces must be allocated to maximize their effectiveness against the predominate threat concentration at the given time...."

1.2 SCOPE AND ORGANIZATION

In Chapter 2 we state and formulate the problem which is the focus of this report. The problem is to maximize the probability of survival of a carrier given air and sub threats, under different scenarios. The solution is the optimum location of the defending ships relative to the carrier; those locations that maximize the survival probability of of the carrier. We build the model; we present the principles, define the parameters and develop the algebraic and probabilistic relationships. We address the analytic limitations of the model.

The purpose of Chapter 3 is to study the behaviour of the model when a single ship is defending the carrier. Our interest lies in discovering the effect on the survival probability of the carrier and on the optimum location of the

ship when we do a parametric analysis. We examine the consequences of changing the sector and increasing the number of targets. We explore the existence of local maxima.

In Chapter 4 we further test the model by considering scenarios in which two and three ships are protecting the carrier. Our experimental studies focus on determining how multiple ships work in conjunction with one another to best defend the carrier. We investigate the conditions under which symmetry exists and consider the result of pure (AAW or ASW threats, but not both) threats. We study the effect of multiple targets, probe the limiting cases, and examine the consequences of changing the threat sectors.

Chapter 5 proposes natural extensions to the model and is aimed at stimulating the mind of the reader to invent his own projects.

We summarize conclusions in Chapter 6.

CHAPTER 2 PROBLEM STATEMENT AND MATHEMATICAL FORMULATION

2.0 SUMMARY

The purpose of this chapter is to define the optimization problem which is the focus of this report, and to develop a mathematical model which captures the essence of the problem. The problem is to maximize the probability of survival of a carrier given air and sub threats, under various scenarios. The solution is the optimum locations of the ships defending the carrier.

The model is a static, probabilistic and two-dimensional representation. The probability that a ship intercepts a target, the optimum intercept distance, the effect of a ship's distance to the carrier and the probability that the carrier survives, are all developed. The limitations of the model are addressed.

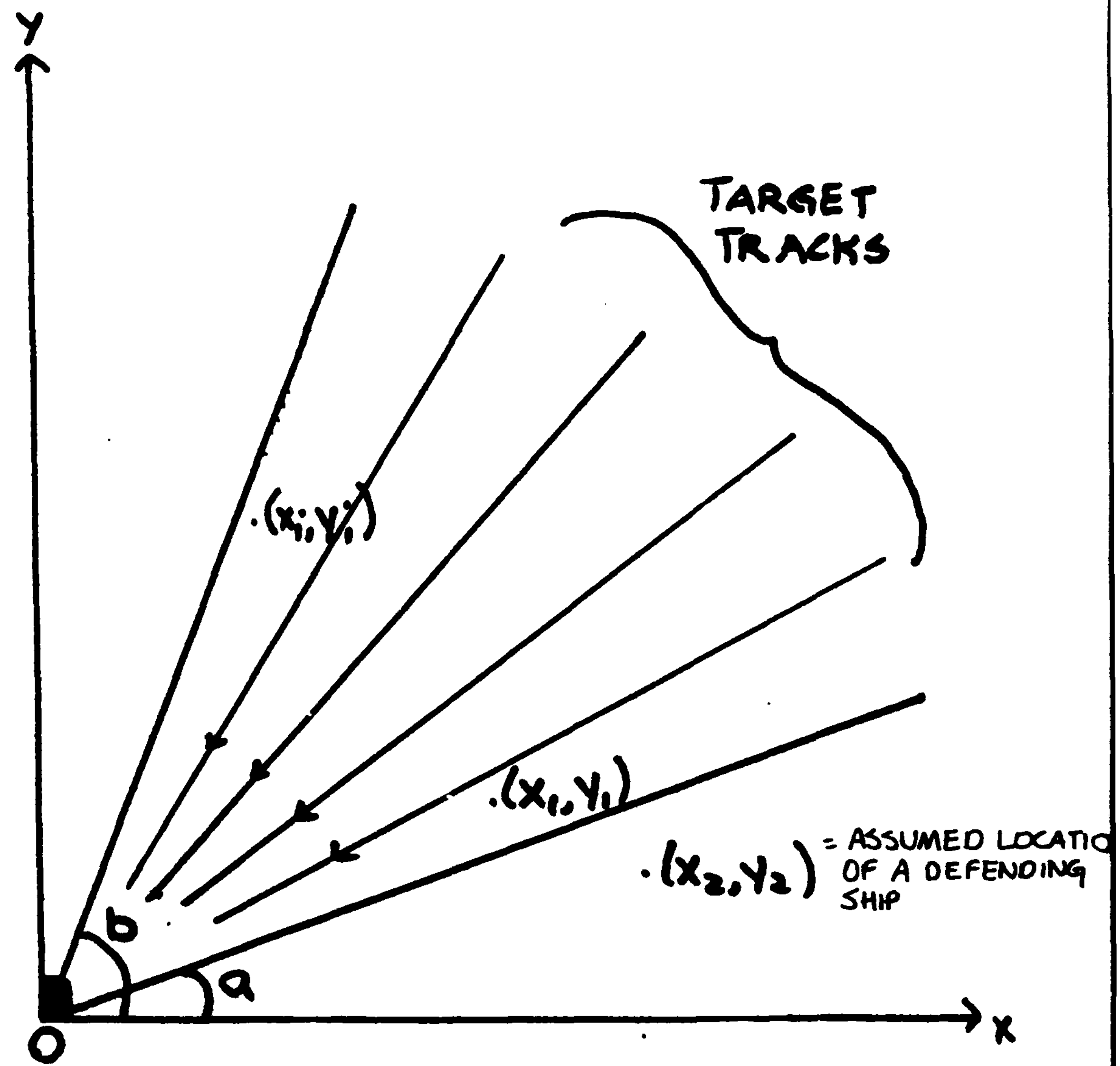
2.1 PROBLEM STATEMENT

The purpose of the problem is to maximize the probability of survival of a carrier given air and sub threats, given simple models of the survival probability.

There are AAW (anti-air warfare) and ASW (anti-sub warfare) ships available to defend the carrier. Missiles (air threats) and submarines (sub threats) are approaching the carrier uniformly over well-defined sectors, which are known with certainty. Each of the AAW and the ASW ships (cruisers and destroyers respectively) have both anti-air and anti-sub capabilities. That is, an AAW ship has superior anti-air warfare, but,is still capable and responsible for intercepting sub targets. Similarly for ASW ships.

The solution to the problem is the optimum location of the ships with respect to the carrier - that location which maximizes the probability of survival of the carrier.

We require that ships not be positioned at the origin (the assumed location of the carrier) since it is assumed that a certain distance is necessary to successfully detect the targets (surveillance). This constraint is incorporated into the mathematical model. Figure 2.0 illustrates this scenario.



<u>Legend</u>

1) Ship i with Coordinates (x_i, y_i)

2) Carrier at the Origin ■

3) Threat Sector (a°, b°)

Figure 2.0 Ships Defending Carrier Against Approaching Targets

2.2 THE MODEL - A MATHEMATICAL FORMULATION

The problem is formulated in 2-dimensions. The carrier is positioned at the origin. Each ship i has Cartesian coordinates (x_i, y_i) which represent its position, relative to the carrier. We assume that threats are approaching the carrier at an angle θ which is uniformly distributed over a known sector.

2.2.1 OPTIMUM INTERCEPTION DISTANCE

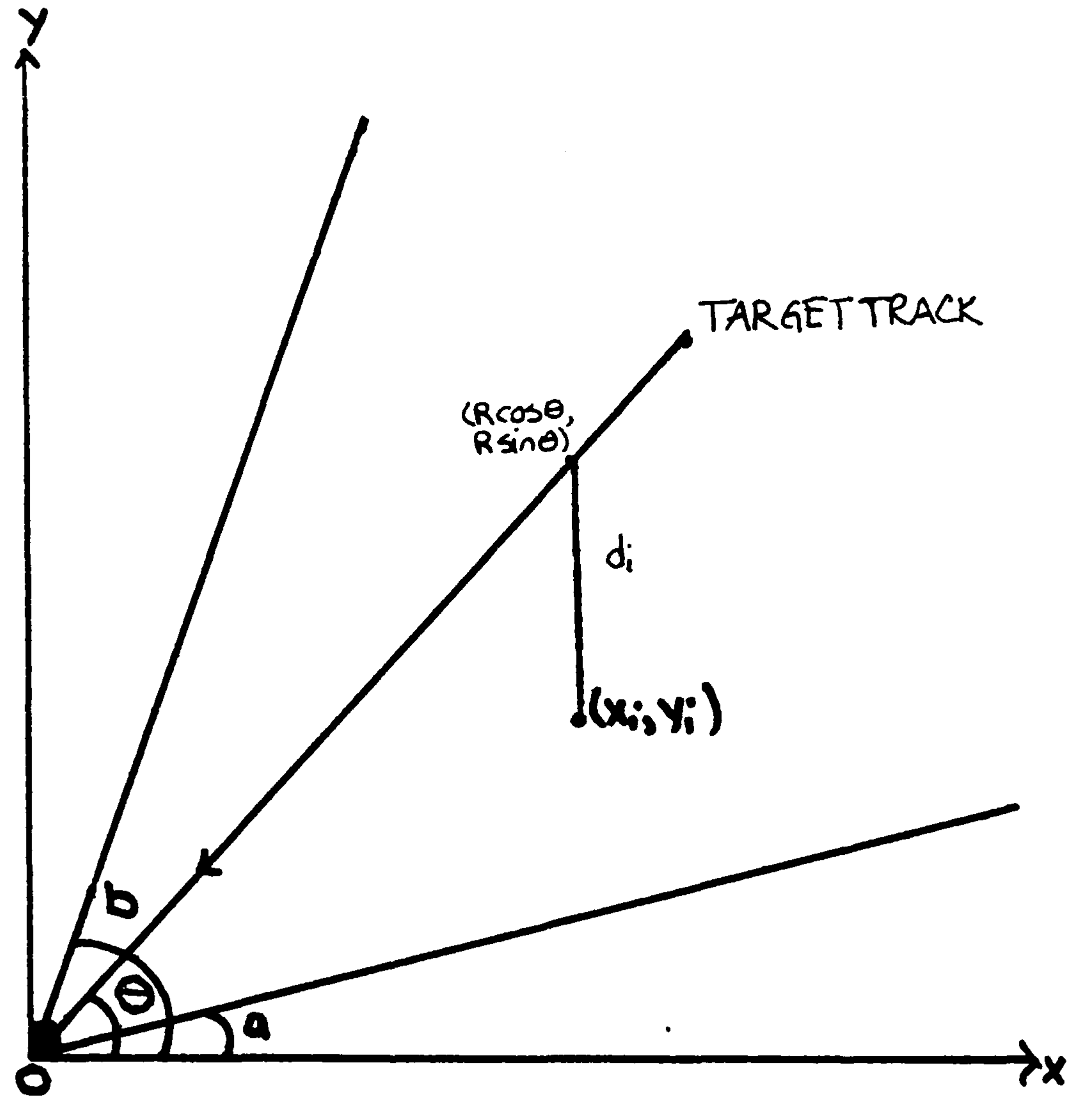
The ship will attempt to intercept a single target when the distance between them is at a minimum. This is the optimum interception distance, and we now establish how it is determined.

Let the (fixed) position of ship i be (x_i, y_i). The target is approaching the carrier at an angle θ, and at any given instant has as its location coordinates $(R\cos\theta, R\sin\theta)$, where R is the current radius (distance) of the target to the carrier. Let d_i be the distance of ship i to the target. We wish to find d_i^*, the optimum interception distance.

The Euclidean distance d_i^2 (see Figure 2.1) is:

$$d_i^2 = (x_i - R\cos\theta)^2 + (y_i - R\sin\theta)^2 \tag{2.1}$$

The optimum distance d_i^* occurs at $\frac{d}{dR}[d_i] = 0$, which yields



<u>Legend</u>

1) Ship i with Coordinates (x_i, y_i)

2) Carrier at the Origin ■

3) Threat Sector (a ,b)

Figure 2.1 Distance from Ship to Target - d_i

$$R = x_i \cos\theta + y_i \sin\theta \tag{2.2}$$

Substituting the value of R in Eq'n (2.2) into Eq'n (2.1), we obtain

$$d^*_i = (x_i \sin\theta) - (y_i \cos\theta) \tag{2.3}$$

which is the optimum distance for ship i to intercept a target.

DEFINITION 2.1: d_i^*

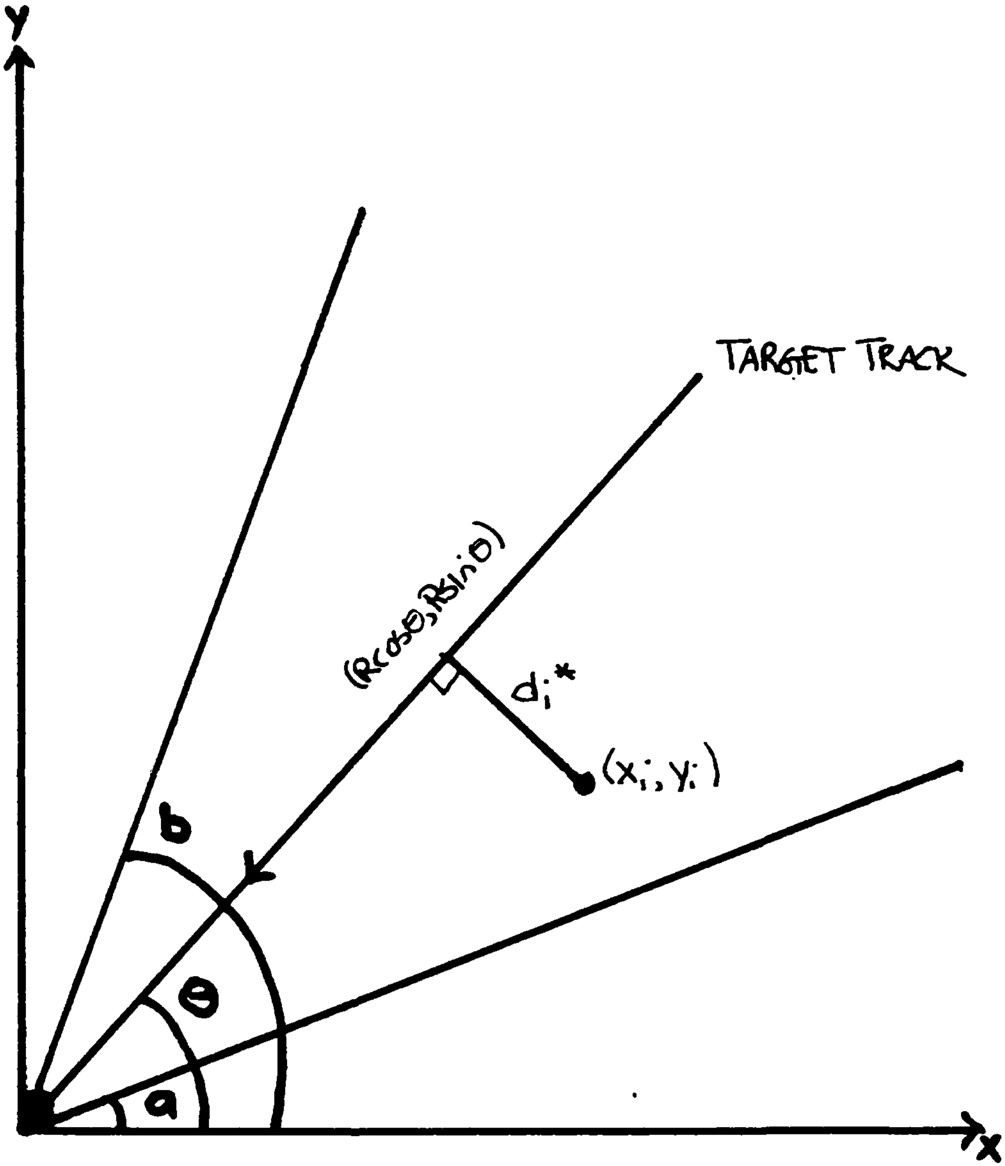
d^*_i is the INTERCEPT DISTANCE and is given by

$$d^*_i = (x_i \sin\theta) - (y_i \cos\theta)$$

Figure 2.2 shows d^*_i .

2.2.2 SOME PROBABILISTIC RELATIONSHIPS

Each cruiser and destroyer has characteristic abilities to kill air and sub targets. These air and sub capabilities are described in terms of probabilities of kill, and we assume that they are a function of both the ship-to-target intercept distance, d^*_i , and the distance r_i of the ship to the carrier, $r_i = \sqrt{x_i^2 + y_i^2}$.



Legend

1) Ship i with Coordinates (x_i, y_i)

2) Carrier at the Origin ■

3) Threat Sector (a ,b)

Figure 2.2 Intercept Distance d_i^*

2.2.2.1 THE EFFECT OF A SHIP'S DISTANCE TO THE CARRIER

Since the carrier is fixed at the origin, all targets of interest pass through the origin.

In Section 2.2.1, we saw that the ship will attempt to kill the target when the distance between them, d_i, is at a minimum. If minimizing this distance were the only objective, then the optimum solution would suggest placing all the ships at the origin, since then d_i^* of Eq'n (2.3) would equal zero. This is clearly infeasible since the ships cannot be on top of the carrier. It is also undesirable since the ships must maintain a certain distance from the carrier in order to detect the targets at a sufficiently large distance from the carrier.

To incorporate this constraint into the problem, we force the probability of intercepting a target to be a function of the ship's distance to the origin. We call this function $P_{r_{A_i}}$. The probability of ship i killing a target increases as its distance to the carrier increases. This prevents the ship from having its final position at the origin.

This assumption, that the further the ship is from the carrier the better is the chance that it will kill a target (for the same intercept distance) appears to make sense. ASW ships, employing passive sonar, cannot be located

near the carrier because of noise interference. For AAW ships we may have similar electromagnetic interference phenomena; also, we want to avoid the battle front being too close to the carrier.

DEFINITION 2.2: $P_{r_{A_i}}$

$P_{r_{A_i}}$, is defined as:

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} r_i)} \quad , \; k_{A_i} > 0 \qquad (2.4)$$

where,

$r_i = \sqrt{x_i^2 + y_i^2}$ is the (Euclidean) distance of ship i to the carrier at the origin, and

k_{A_i} is a parameter associated with each cruiser and destroyer.

Figure 2.3 shows the shape of $P_{r_{A_i}}$.

The value of k_{A_i} selected affects the optimal position of the ships - a smaller k_{A_i} value results in a ship being farther from the carrier. k_{A_i} is chosen so as to place the ships' distances from the carrier within a reasonable range (i.e. 20 - 100 miles). In addition, the k_{A_i} value for destroyers

is lower than the one for cruisers because the destroyers must be located farther from the carrier for ASW surveillance reasons in order to detect and kill submarines.

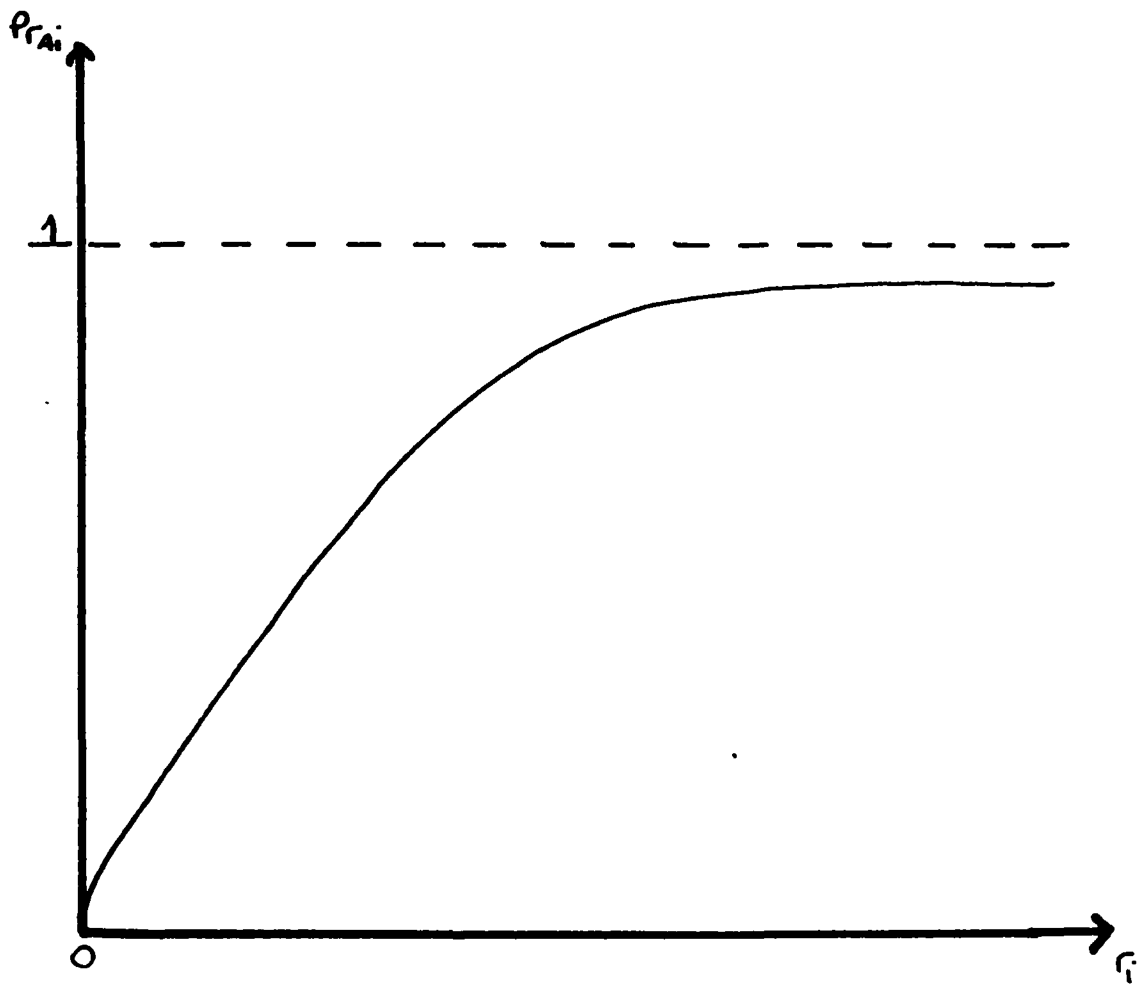


Figure 2.3 Functional Form of $P_{r_{A_i}}$ as a Function of r_i, the ith Ship's Distance from the Carrier

$P_{r_{A_i}}$ tends to 1 as r_i , the distance from the ship to the carrier, tends to infinity. It's smallest value is "0" and is attained when the distance from ship i to the carrier is zero. This is how we use $P_{r_{A_i}}$ in our model to force the ships away from the origin.

2.2.2.2 PROBABILITY OF INTERCEPTION

Each ship i has two functions which fully describe its ability to to intercept targets. P_{A_i} of Eq'n (2.5) reflects ship i's ability to intercept AAW targets. An analogous function, P_{S_i} , reflects ship i's ASW intercept capability.

P_{A_i} and P_{S_i} are probabilites since their values lie in (0,1). However, they are not probability distributions since they do not sum to unity as they are not scaled accordingly .

Every ship i has its own range for killing both AAW and ASW targets. This suggests the following definitions.

DEFINITION 2.3: Σ_{A_i} , Σ_{S_i}

Σ_{A_i} is called the AAW RANGE FACTOR of ship i and represents ship i's ability to kill AAW targets.

Σ_{S_i} is called the ASW RANGE FACTOR of ship i and represents ship i's ability to kill ASW targets.

DEFINITION 2.4: P_{A_i}

P_{A_i}, the PROBABILITY(SHIP I KILLS AN AAW TARGET), is

$$P_{A_i} = P_{r_{A_i}} \; e^{(-d_i^{*2}/\Sigma_{A_i})} \qquad (2.5)$$

where,

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} r_i)}, \; k_{A_i} > 0 \qquad (2.4)$$

$$d_i^* = (x_i \sin\theta - y_i \cos\theta) \qquad (2.3)$$

and,

Σ_{A_i} is the range AAW range factor.

Figure 2.4 shows P_{A_i} .

We see that P_{A_i} , the probability that ship i kills an AAW target, decreases exponentially with the distance d_i^{*2} . In addition, the probability of kill

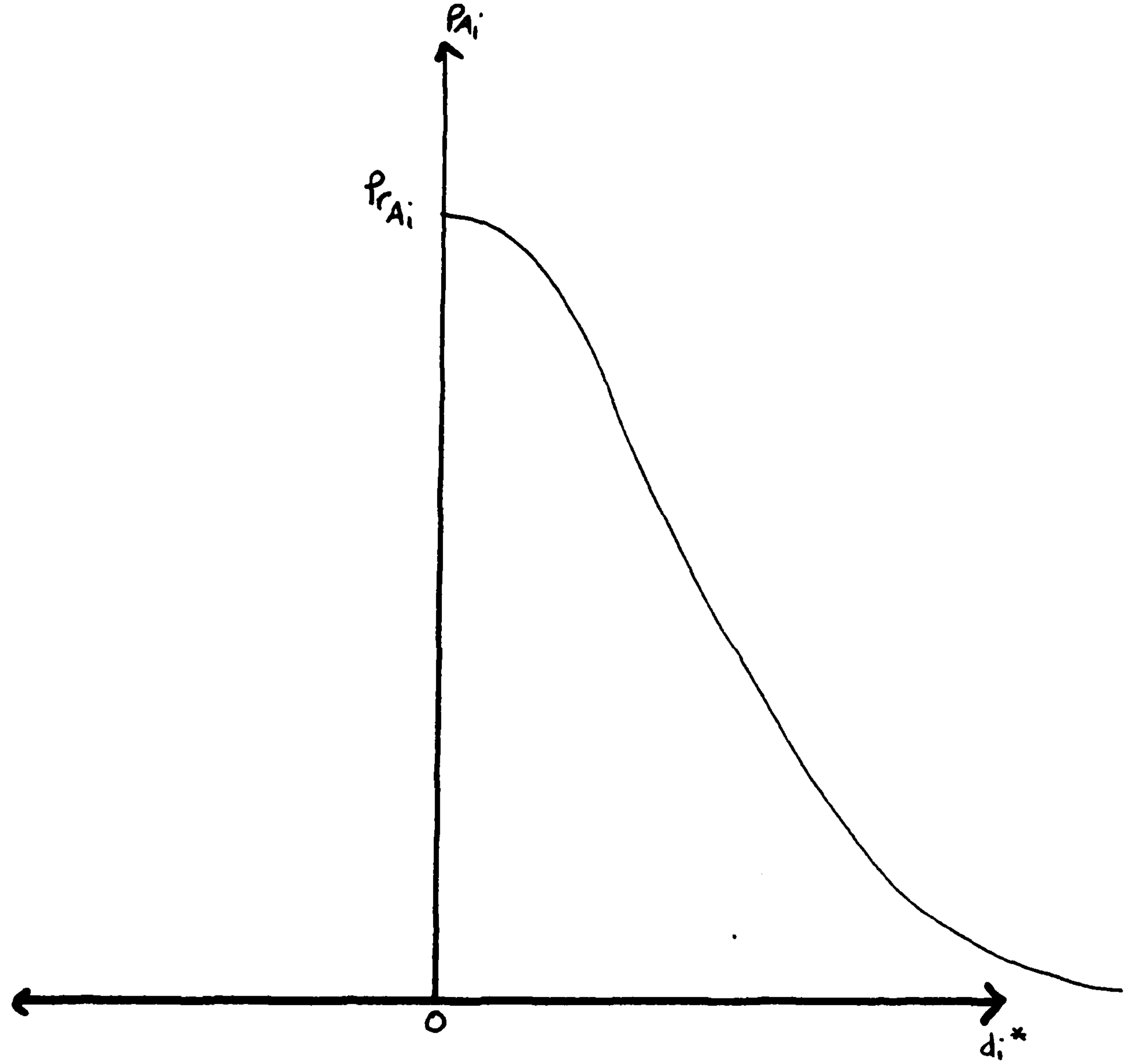


Figure 2.4 Functional Form of the Kill Probability as a Function of the Optimum Intercept Distance d_i^*

increases as r_i , the distance from the ship to the carrier increases. A larger Σ_{A_i} value is favourable and yields a larger P_{A_i} .

Similarly, for every ship i we have a function which completely describes its ability to intercept ASW targets. We call this function P_{S_i} .

DEFINITION 2.5: P_{S_i}

P_{S_i} , the PROBABILITY(SHIP I KILLS AN ASW TARGET), is

$$P_{S_i} = P_{r_{S_i}} \; e^{(-d_i^{*2} / \Sigma_{S_i})} \qquad (2.6)$$

where,

$$P_{r_{S_i}} = 1 - e^{-(k_{S_i} r_i)} \qquad (2.4)$$

$$d_i^* = (x_i \sin\theta - y_i \cos\theta) \qquad (2.3)$$

and,

Σ_{S_i} is the ASW range factor.

P_{S_i} has exactly the same properties as P_{A_i} .

2.2.2.3 PROBABILITY DISTRIBUTION OF THE TARGET

The targets are approaching the carrier at an angle θ which we assume is uniformly distributed over a given sector. The sectors are described in angles. Let $f(\theta)$ be the probability density function (p.d.f.) of a single target. Then $f(\theta)$ is given by:

$$f(\theta) = 1/(b - a) \qquad a \leq \theta \leq b$$
$$= 0 \qquad \text{otherwise}$$

Figure 2.5 shows $f(\theta)$.

2.2.3 SURVIVAL PROBABILITY OF THE CARRIER

We now have all the information required to construct the probability function associated with the survival of the carrier. We initially consider the case of a single ship defending the carrier, and a single target attacking the carrier, and later expand to include the possibility of multi-ship protection and multi-target, multi-sector attacks. We assume throughout that the carrier has no defensive capabilities of its own.

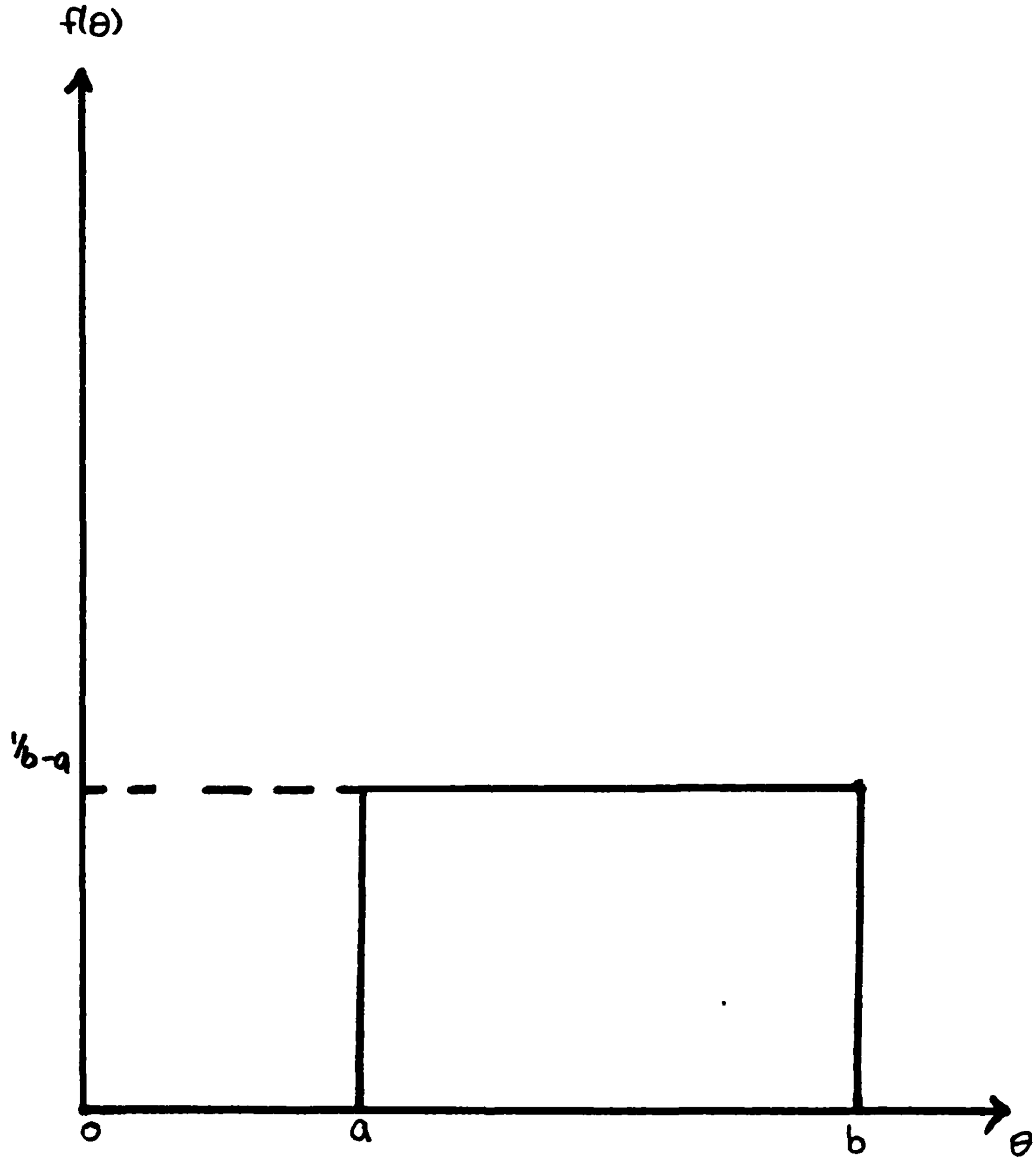


Figure 2.5 Uniform P.D.F. of Target over Sector $a \leq \theta < b$

2.2.3.1 PERMISSIBLE SECTORS AND LIMITATIONS OF THE MODEL

In this section, we present two hypothetical scenarios based on two different sectors. We show how the expression for the probability of survival of the carrier changes when different types of sectors are used.

Case 1a: 1 Ship, 1 Target, 1 Sector

We formulate the problem in terms of an AAW threat, i.e. an AAW target is approaching the carrier at an angle θ that is uniformly distributed over a sector defined by (a,b). All calculations for an ASW target are analogous. Consider the scenario pictured in Figure 2.6.

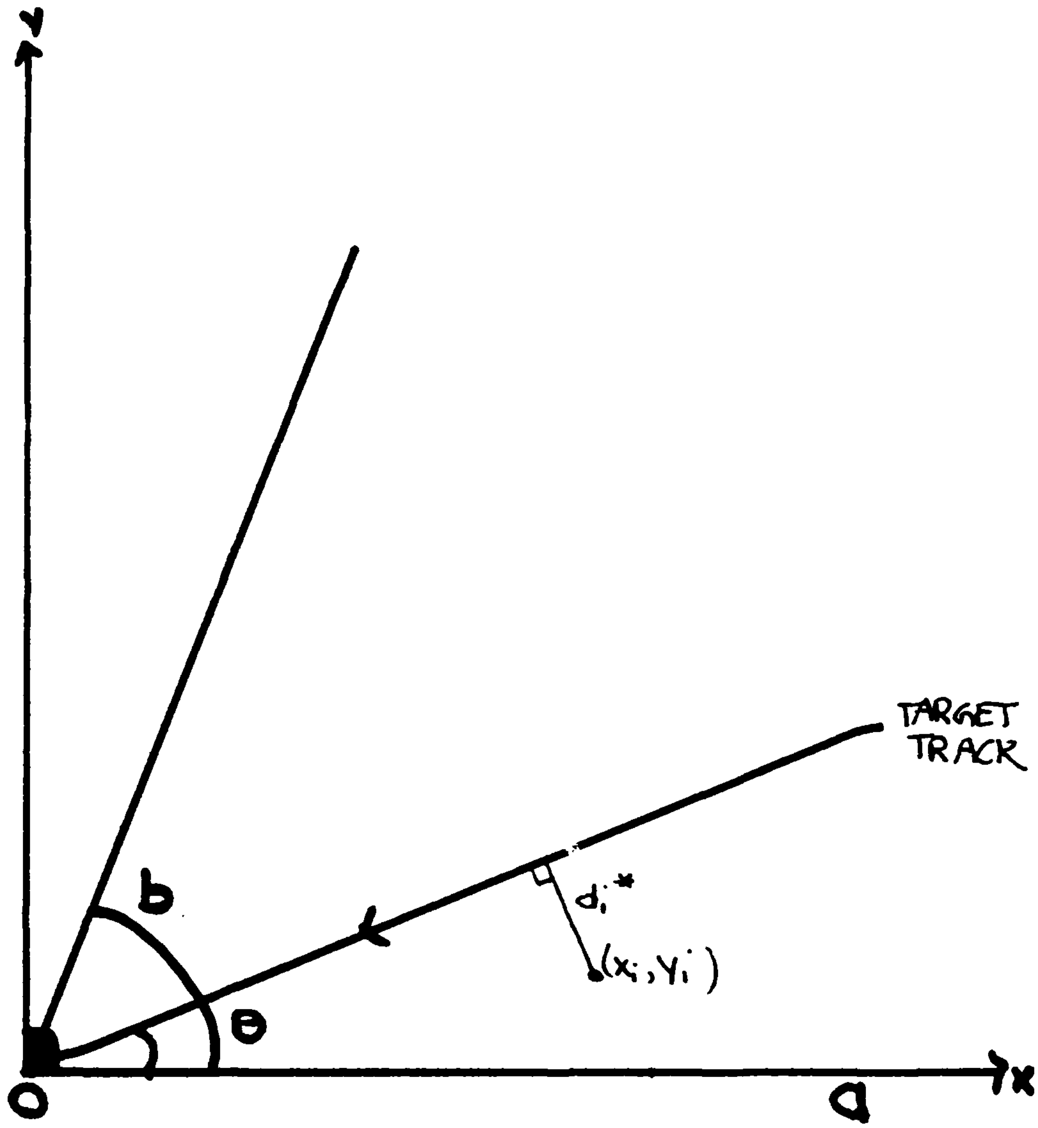
For this scenario we have that:

$$\text{Prob(ship i kills an AAW target)} = P_{A_i} \qquad (2.5)$$

so that ,

$$\text{Prob(ship i kills an AAW target over sector (a,b))}$$
$$= \int_a^b f(\theta) \times P_{A_i} \, d\theta \qquad (2.7)$$

This implies the following about the survival probability of the carrier.



<u>Legend</u>

1) (x_i, y_i): Ship i with Coordinates (x_i, y_i)

2) Carrier at the Origin ■

3) Threat Sector (a, b), $a = 0$

Figure 2.6 Case 1a: 1 Ship, 1 Target, 1 Sector

Prob(carrier survives) = Prob(the ship kills the AAW target)

$$= \int_a^b f(\theta) \times P_{A_i} \, d\theta \qquad (2.7)$$

Figure 2.7 shows the area of interest of the survival probability of the carrier.

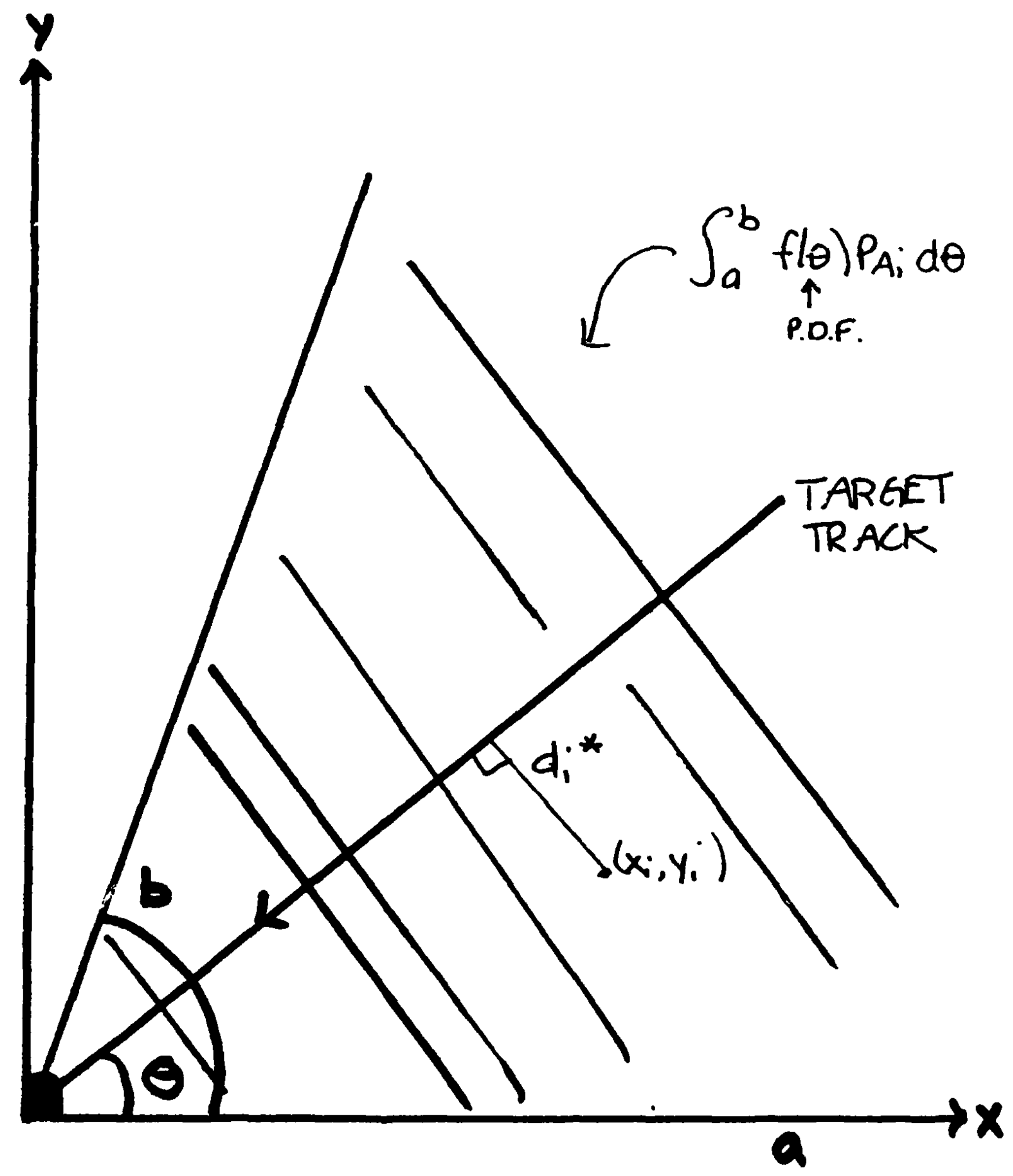
We now establish the survival probability of the carrier under a slightly different scenario.

Case 1b: 1 Ship, 1 Target, 1 Sector

Now consider the same problem as Case 1a but with a different threat sector, as shown in Figure 2.8.

If we divide the sector (a,b) into two sectors (a,c) and (c,b), we notice that Eq'n (2.7) is not valid in (c,b). This is so, because for any target in this sector, d^*_i , the intercept distance, is such that the ship will intercept the target after it has passed through the origin - i.e. after it has passed the carrier. This fact is illustrated in Figure 2.9.

This does not make sense - there is no reason to kill targets after after they have passed the carrier. In this situation, the best that the ship can do is to kill the target as it is passing through the origin. In other words, to kill the target at a distance d^*_i such that:



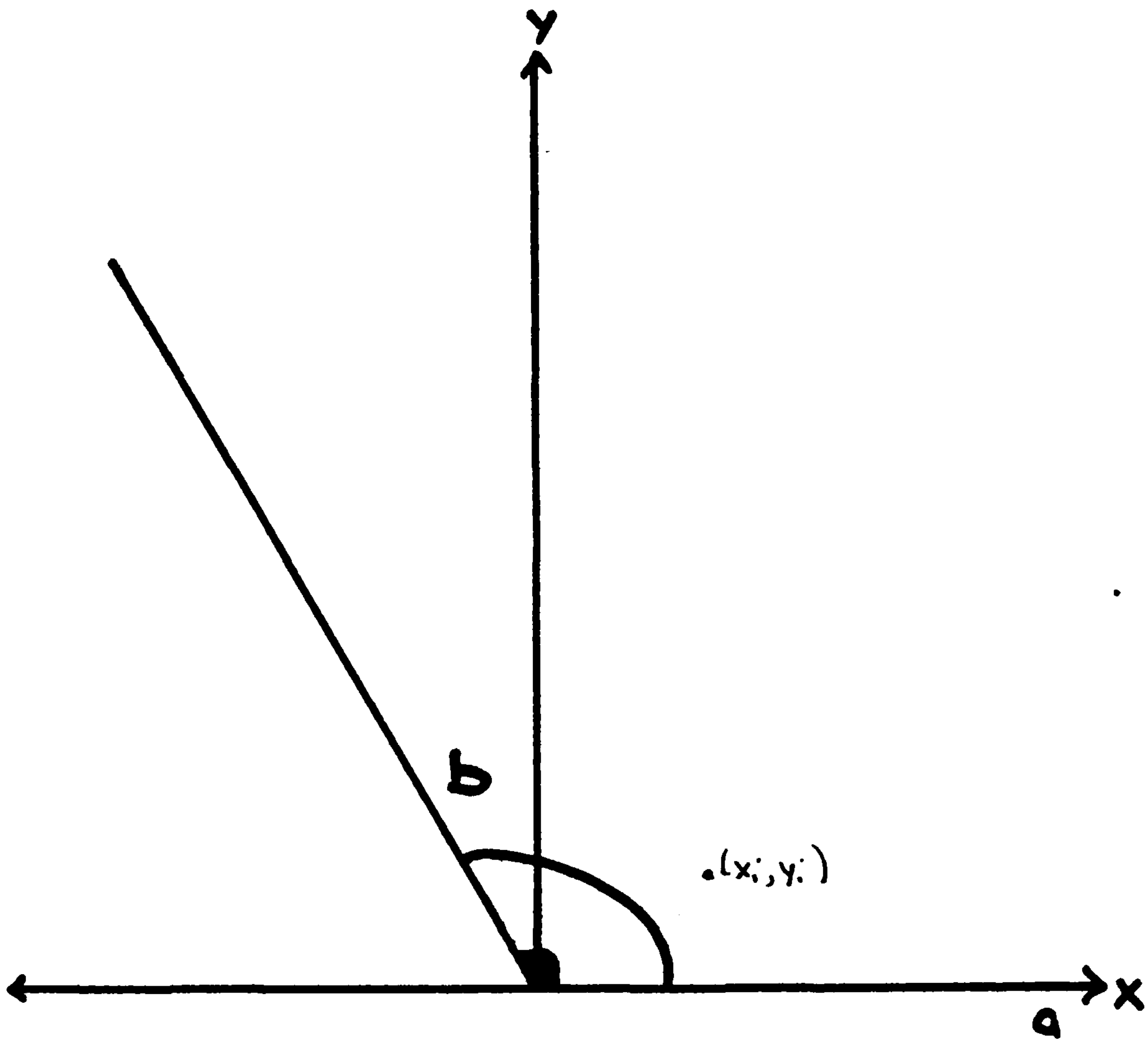
<u>Legend</u>

1) Ship with Coordinates (x_i, y_i)

2) Carrier at the Origin ■

3) Threat Sector (a ,b)

Figure 2.7 Area of Interest in Survival Probability



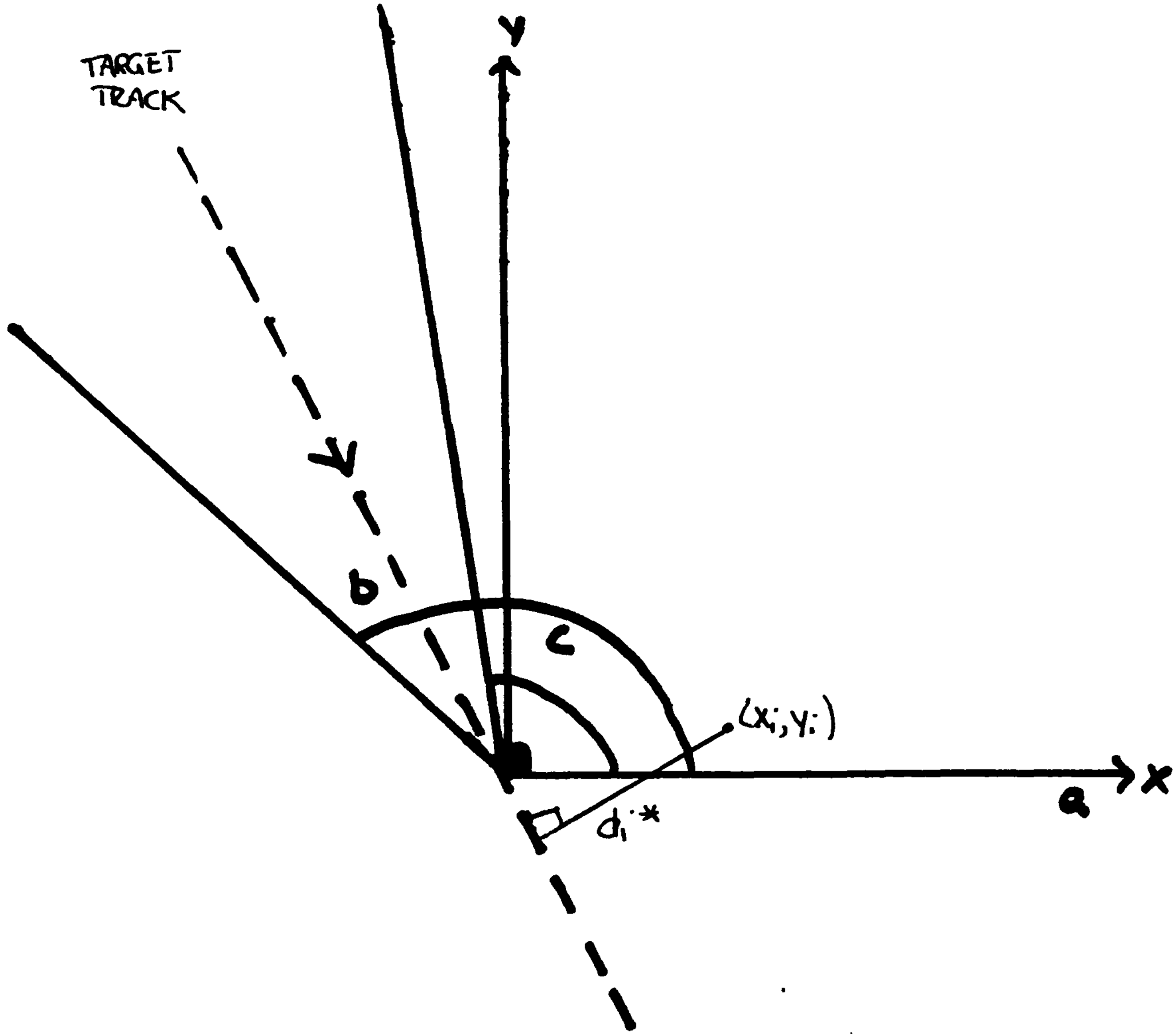
<u>Legend</u>

1) Ship with Coordinates (x_i, y_i)

2) Carrier at the Origin ■

3) Threat Sector (a ,b), a = 0

Figure 2.8 Case 1b



<u>Legend</u>

1) Ship with Coordinates (x_i, y_i)

2) Carrier at the Origin

3) Threat Sector (a ,b) Divided into Two Sectors, (a ,c) and (c ,b)

Figure 2.9 Case 1b: Divided Sector

$$d^*_i = r_i$$

where $r_i = \sqrt{x_i^2 + y_i^2}$ is the distance of ship i to the carrier.

Figure 2.10 illustrates the result of $d^*_i = r_i$.

In order to include the possibility of a Case 1a or a Case 1b sector we generalize d^*_i .

DEFINITION 2.6: ϕ_i

For any ship i with coordinates (x_i, y_i), $\phi_i = \tan(y_i/x_i)$ is the angle of ship i with respect to the origin.

DEFINITION 2.7: ϕ_1, ϕ_2

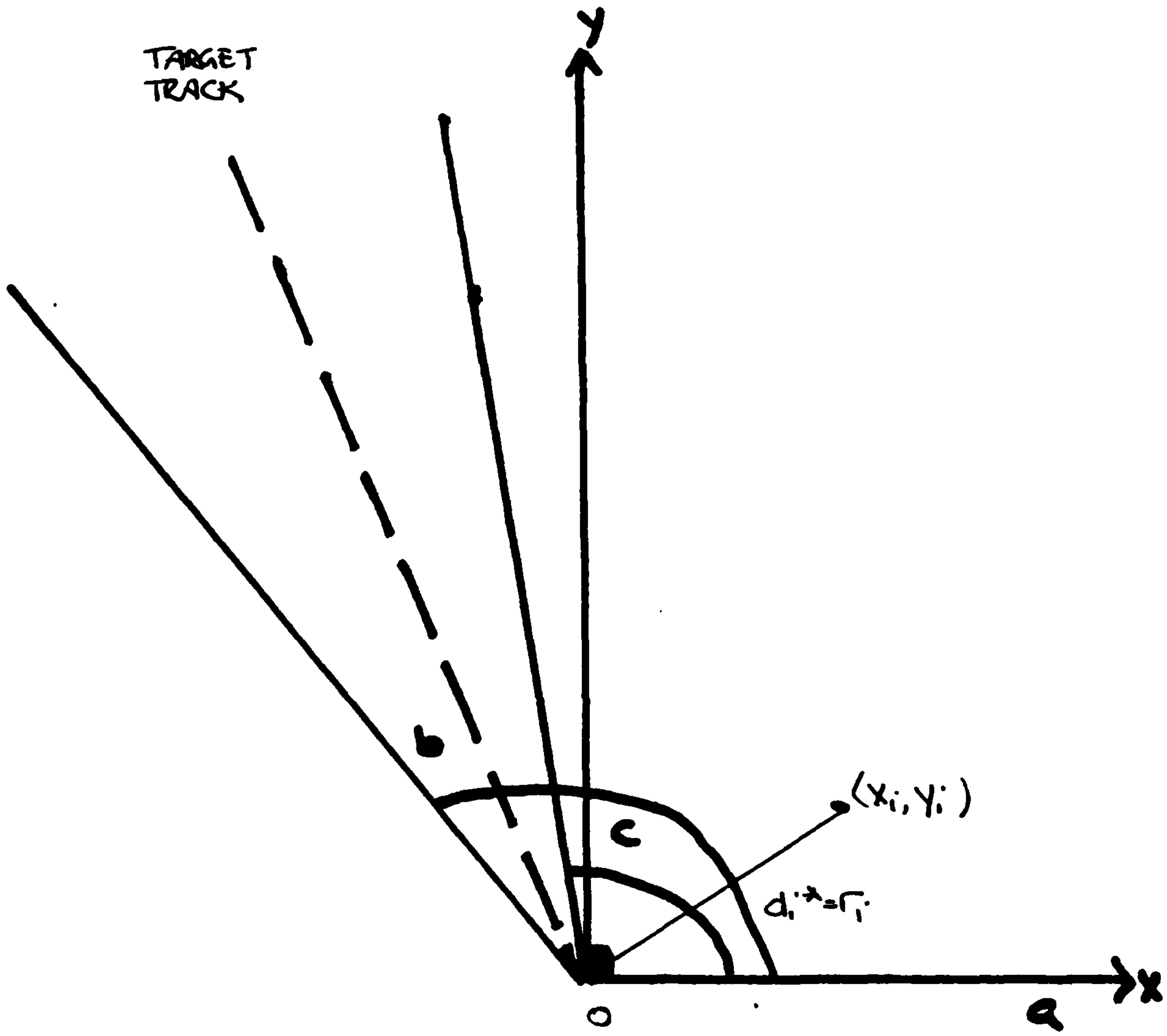
Let θ be the angle of approach of a target. Then θ defines two angles, ϕ_1 and ϕ_2 . They are:

$$\phi_1 = \theta - \pi/2 \text{ and,}$$

$$\phi_2 = \theta + \pi/2.$$

Examples of ϕ_1 and ϕ_2 are illustrated in Figure 2.11.

For any angle θ and any ship i with coordinates (x_i, y_i), d^*_i is the shortest and therefore the perpendicular distance from ship i to the target. In other words, $d^*_i = x_i \sin\theta - y_i \cos\theta$ if and only if $\phi_1 \leq \phi_i \leq \phi_2$. This corresponds to



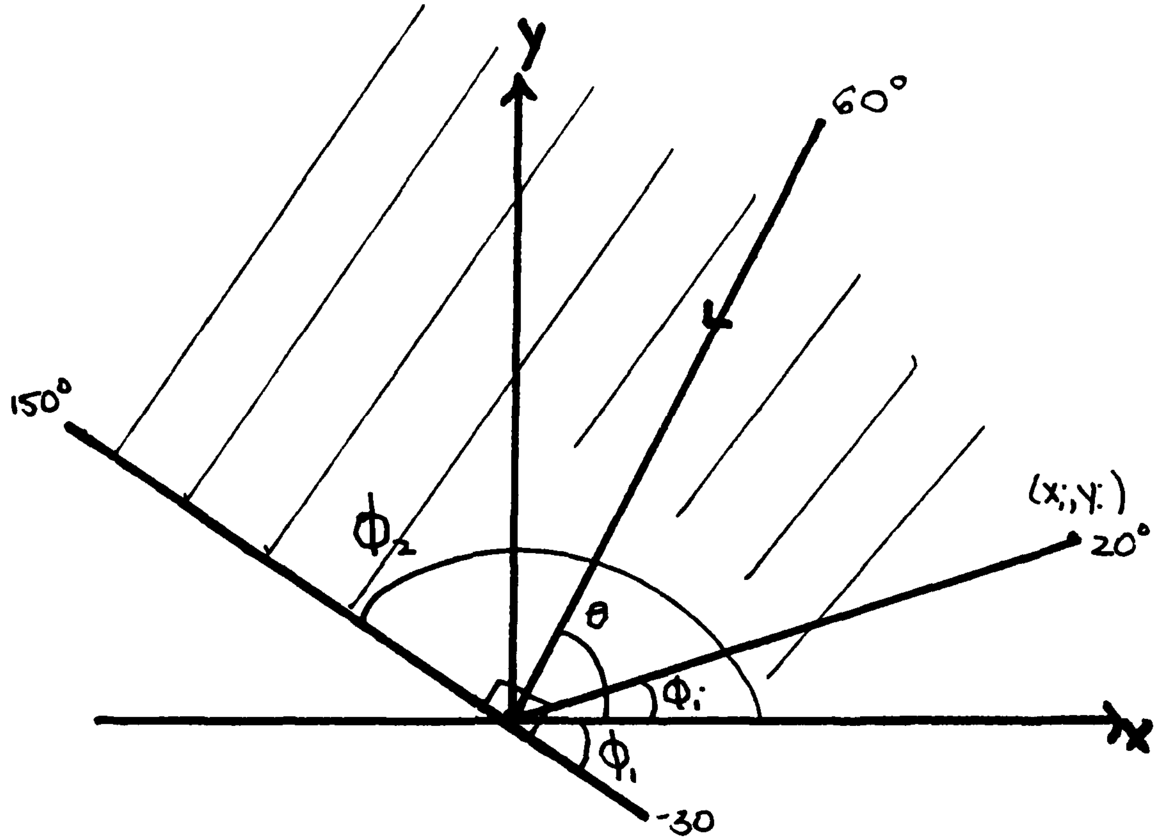
<u>Legend</u>

1) Ship with Coordinates (x_i, y_i)

2) Carrier at the Origin ■

3) Threat Sector (a ,b), a = 0

Figure 2.10 Case 1b: Killing Target as it Passes through the Origin



<u>Legend</u>

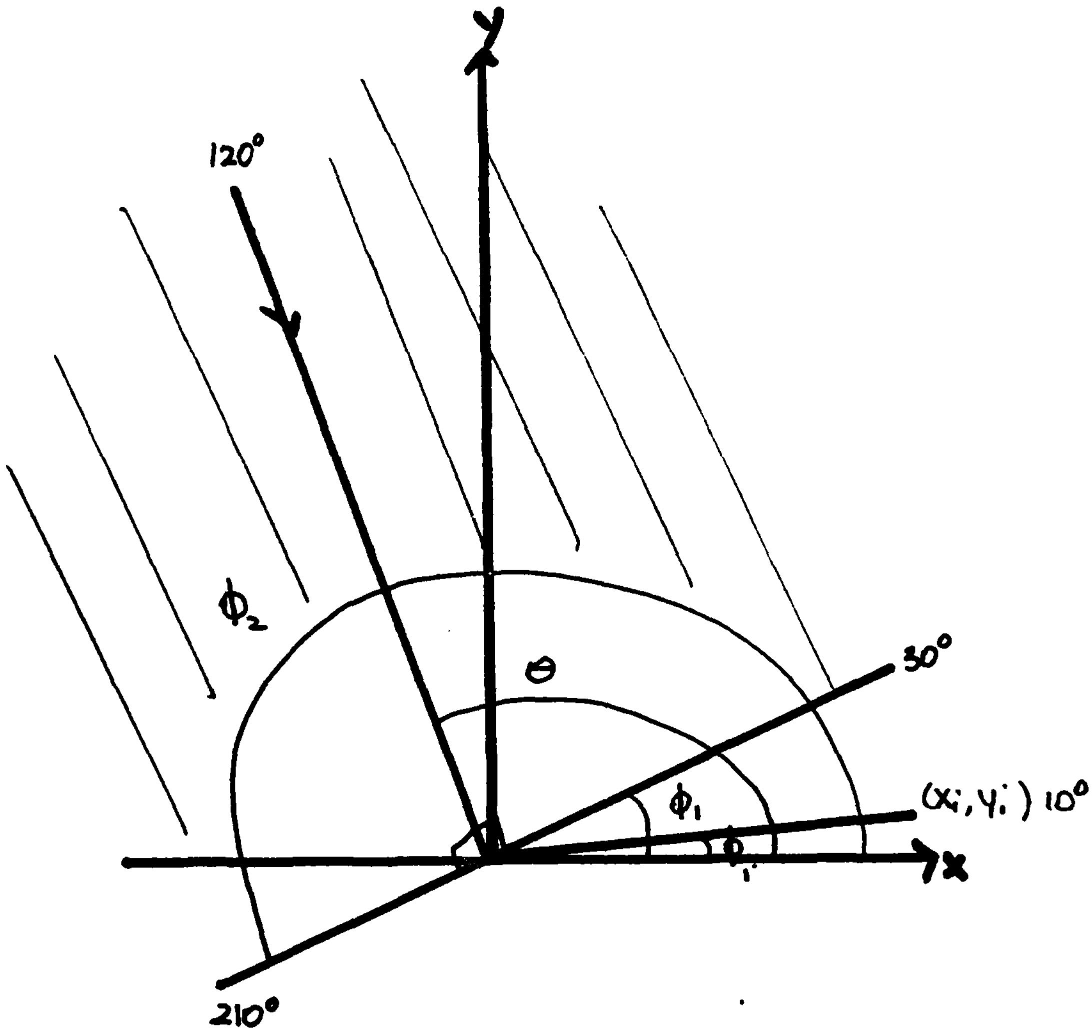
1) θ = Angle of Target = 60

2) ϕ_i = Angle of Ship i = 20

3) $\phi_1 = \theta - \pi/2 = -30$

4) $\phi_2 = \theta + \pi/2 = 150$

Figure 2.11a Example of ϕ_1 and ϕ_2



<u>Legend</u>

1) θ = Angle of Target = 120

2) ϕ_i = Angle of Ship i = 10

3) $\phi_1 = \theta - \pi/2 = 30$

4) $\phi_2 = \theta + \pi/2 = 210$

Figure 2.11b Example of ϕ_1 and ϕ_2

saying that the ship must be within the shaded regions of the examples of Figure 2.11. This is expressed more formally in Definition 2.8.

DEFINITION 2.8: GENERALIZED d^*_i

d^*_i is the INTERCEPT DISTANCE if and only if

$$d^*_i = x_i \sin\theta - y_i \cos\theta \qquad \phi_1 \le \phi_i \le \phi_2$$

$$= r_i \qquad \text{otherwise}$$

For the scenario in Figure 2.10 we conclude that:

$$\text{Prob(ship i kills the AAW target)} = \int_a^c f(\theta) \times P_{r_{A_i}} \times e^{\frac{-(x_i \sin\theta - y_i \cos\theta)^2}{\Sigma A_i}} d\theta \tag{2.8}$$

$$+ \int_c^b f(\theta) \times P_{r_{A_i}} \times e^{\frac{-r_i^2}{\Sigma A_i}} d\theta$$

In this report, we wish to examine only scenarios resulting from the type of sector found in Case 1a. This leads to the following definition.

DEFINITION 2.9: PERMISSIBLE SECTOR

For a given ship i with coordinates (x_i, y_i), a sector defined by (a, b) is called a PERMISSIBLE sector if and only if

$$d^*_i = x_i \sin\theta - y_i \cos\theta \qquad \text{for all } \theta,\ a \leq \theta \leq b$$

COMMENT ON PERMISSIBLE SECTORS:

Whether or not a sector is permissible depends on both the position of the ship(s) as well as the nature of the sector (i.e. size and location). An example of a permissible sector is given in Figure 2.12a. The same sector with ship i in another position is not permissible and is shown in Figure 2.12b. From now on we study only permissible sectors.

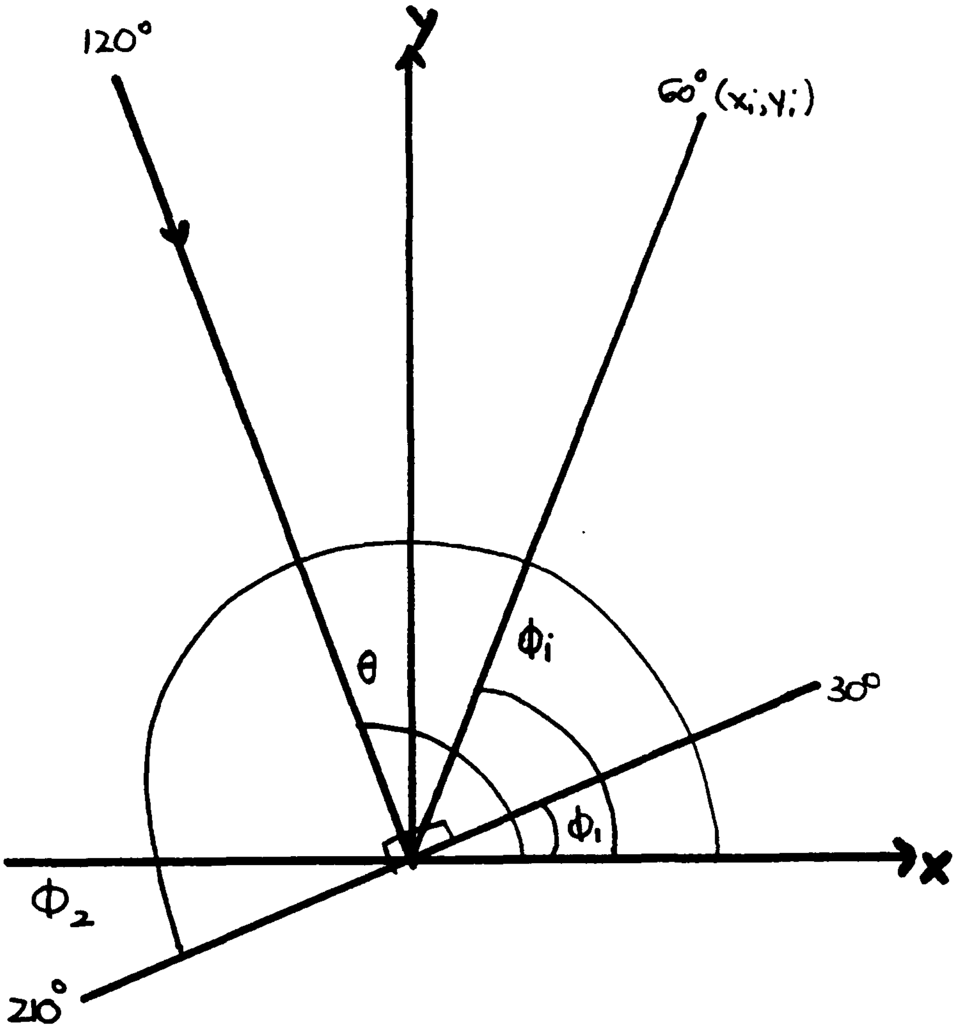
Case 2: M Ships, 1 Target, 1 Threat Sector

We return to the case of permissible sectors and consider Figure 2.13. For the case of M ships defending the carrier , we obtain the following expression:

Prob(carrier survives) = Prob(at least one ship kills the AAW target)

= 1 - Prob(no ship kills the AAW target)

$$= \int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i})) \quad d\theta \qquad (2.9)$$



Legend

1) Threat Sector (0 ,120)

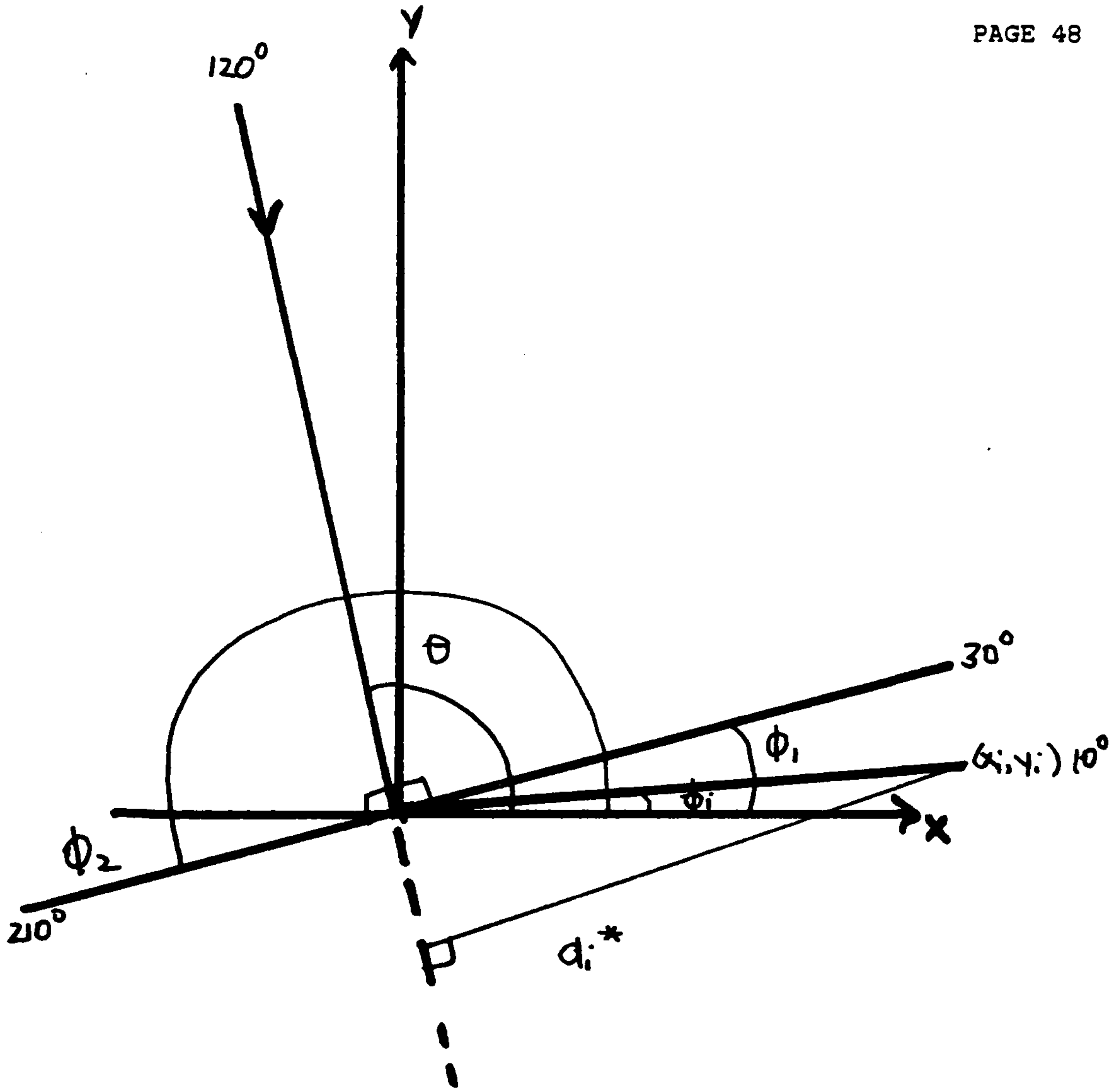
2) θ = Angle of Target = 120

3) ϕ_i= Angle of Ship i = 60

4) $\phi_1 = \theta - \pi/2 = 30$

5) $\phi_2 = \theta + \pi/2 = 210$

Figure 2.12a Example of Permissible Sector, $\phi_1 \leq \phi_i \leq \phi_2$



<u>Legend</u>

1) Threat Sector (0 ,120)

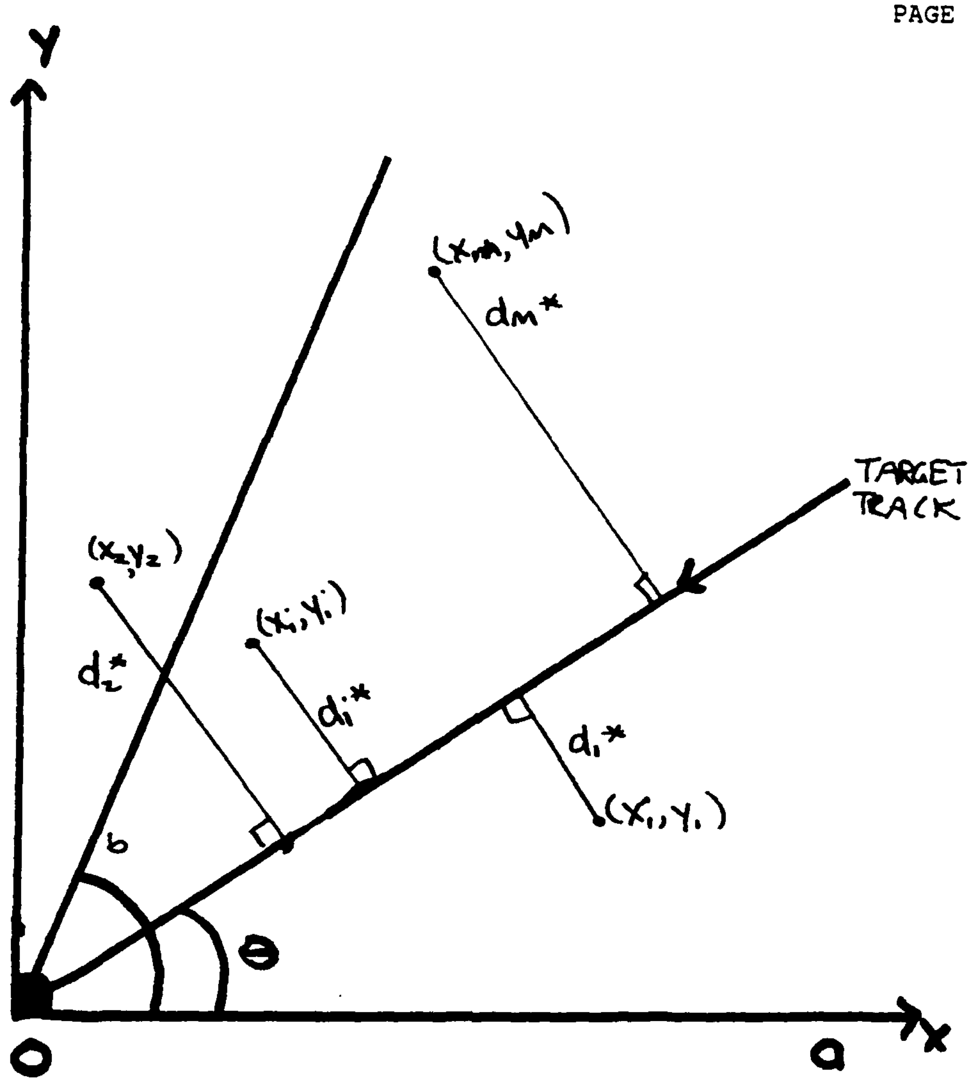
2) θ = Angle of Target = 120

3) ϕ_i = Angle of Ship i = 10

4) $\phi_1 = \theta - \pi/2 = 30$

5) $\phi_2 = \theta + \pi/2 = 210$

Figure 2.12b Example of Non-Permissible Sector



Legend

1) Ship with Coordinates (x_i, y_i)

2) Carrier at the Origin

3) Threat Sector (a ,b)

4) d^*_i = Intercept Distance

Figure 2.13 Case 2: M Ships, 1 Target, 1 Sector

Thus, we assume that each of the M ships will attempt to intercept the target independently.

Case 3: M Ships, N Independent and Identically Distributed Targets, 1 Threat Sector

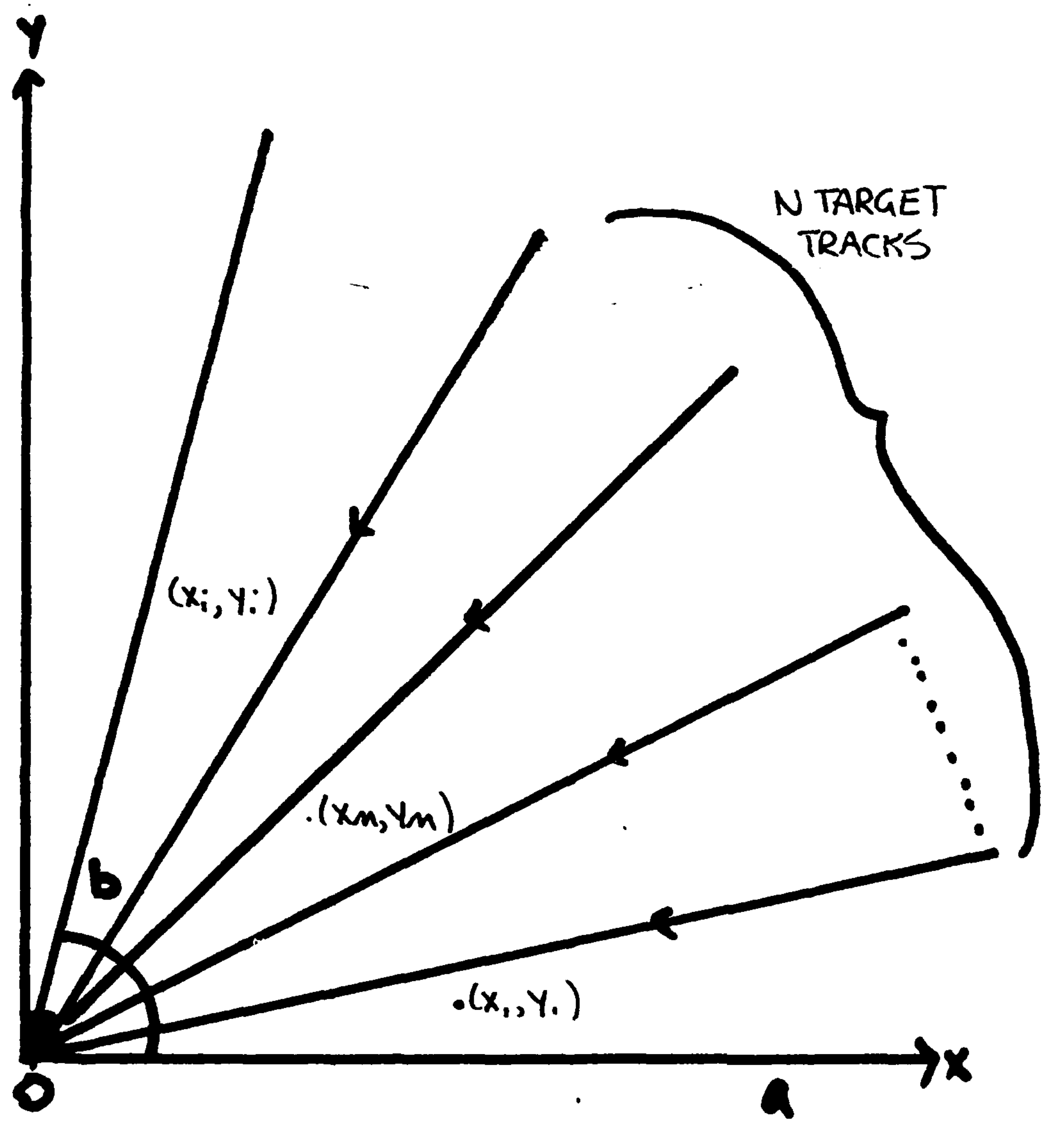
Targets approach the carrier independent of each other. It is, therefore, easy to generalize this problem for a situation where the carrier is threatened by more than one target. We have then, for the case of N targets:

$$\text{Prob(carrier survives)} = \left\{ \int_a^b f(\theta) \quad (1 - \prod_{i=1}^{m}(1-P_{A_i})) \, d\theta \right\}^N \qquad (2.10)$$

Figure 2.14 shows the situation in Case 3.

Case 4: M Ships, N Independent and Identically Distributed AAW Targets, Q Independent and Indentically Distributed ASW Targets, 2 Identical Threat Sectors (1 AAW; 1 ASW)

Finally, we include the possibility of both kinds of threats - AAW and ASW. Figure 2.15 pictures the situation in Case 4.



<u>Legend</u>

1) Ship i with Coordinates (x_i ,y_i)

2) Carrier at the Origin ■

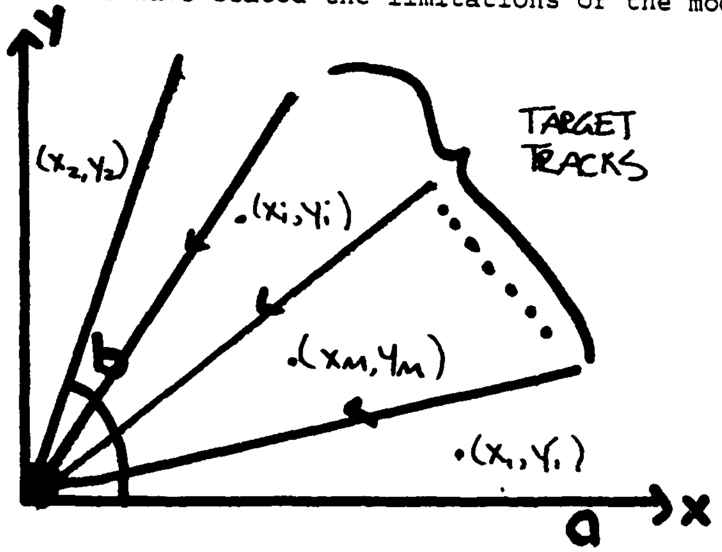
3) Threat Sector (a ,b), a = 0

Figure 2.14 Case 3: M Ships, N Targets, 1 Sector

$$\text{Prob(carrier survives)} = \left\{ \int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \, d\theta \right\}^N$$
$$\times \left\{ \int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{S_i} \quad)) \, d\theta \right\}^Q \qquad (2.11)$$

2.3 CONCLUDING REMARKS

In this chapter, we have defined all the necessary carrier survival probabilities for our model, and we have stated the limitations of the model.



Legend

1) Ship i with Coordinates (x_i, y_i)
2) Carrier at the Origin
3) Two Identical Threat Sectors: AAW in (a ,b) and ASW in (a ,b)

Figure 2.15 Case 4: M Ships, N AAW Targets, Q ASW Targets, Two Identical Threat Sectors (One AAW, One ASW)

CHAPTER 3 SINGLE SHIP - SINGLE SECTOR AND DUAL CAPABILITIES CASE STUDIES

3.0 SUMMARY

The purpose of this chapter is to explain, with numerical experimental evidence, the properties of the mathematical model developed in Chapter 2. The experiments are restricted to a scenario in which only one ship is defending the carrier. In the first part of this chapter, the ship is characterized by the fact that it has only AAW (or ASW) capabilities. In the remaining part of the chapter, the problem and the experiments are expanded to include a ship with dual (i.e. AAW and ASW) capabilities, so that it can defend the carrier against both types of threats.

We study the effects on the survival probability of the carrier and on the optimum location of the ship when the size of the threat sector is changed and when we introduce the possibility of multiple targets. This is all accomplished through the use of a numerical optimization program. Unfortunately, even under the simplest circumstances, the optimal ship location problem cannot be solved analytically by closed-form formulae.

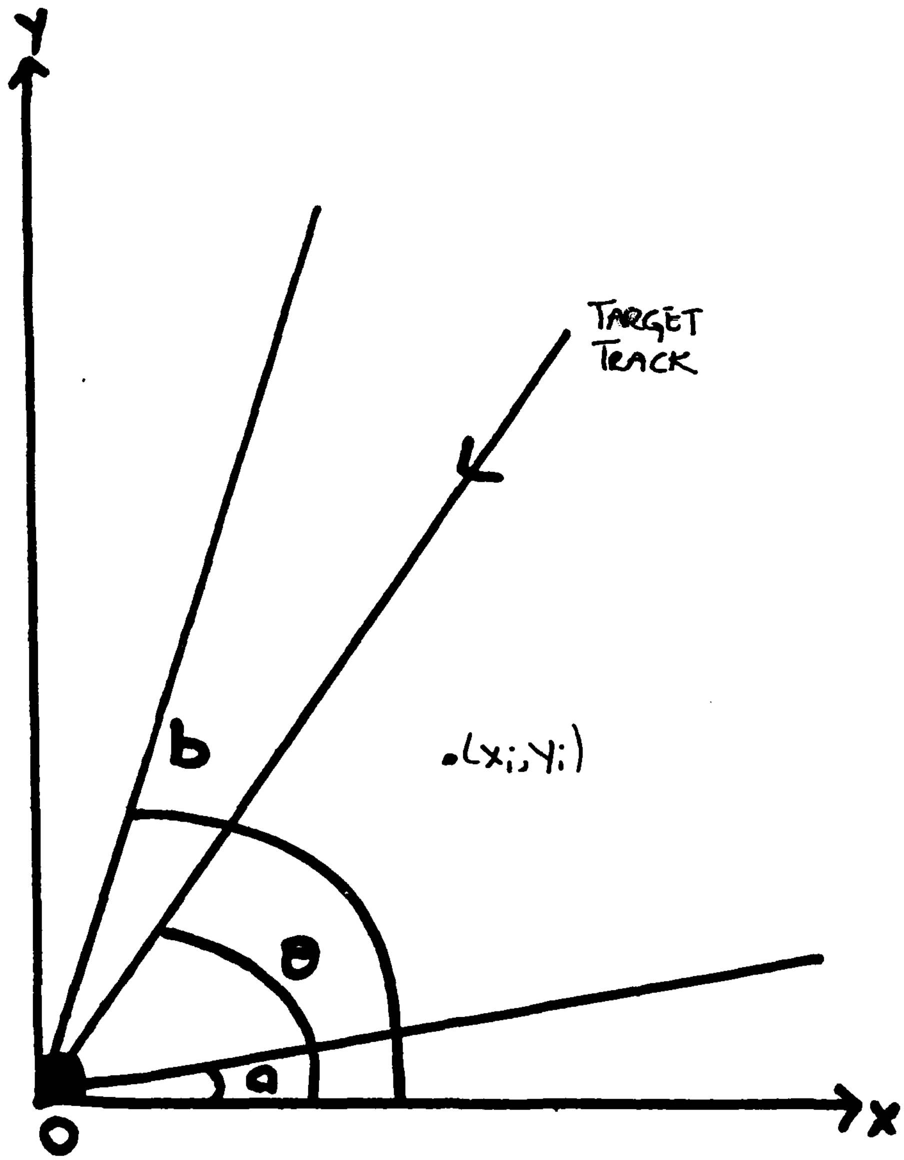
We explore the existence of local maxima. We draw conclusions about the behaviour of the model and set the stage for the experiments of Chapter 4 - the multiple ship case.

3.1 PROBLEM STATEMENT - SINGLE SECTOR, SINGLE WARFARE CAPABILITY

The purpose of the problem is to maximize the probability of survival of a carrier given a single air or sub threat. The threat is uniformly distributed over a known sector. A single AAW or ASW ship is defending the carrier. The solution to the problem is the optimum location of the AAW or ASW ship with respect to the carrier; that location which maximizes the survival probability of the carrier. We formulate the problem in terms of an AAW ship; the formulation for an ASW ship is analogous.

The purpose of this chapter is to perform parametric analyses; to study the quantitative impact of different parameter values in our model on the survival probability of the carrier and on the optimum location of the ship.

3.2 MATHEMATICAL FORMULATION - SINGLE SECTOR, SINGLE CAPABILITY



<u>Legend</u>

(1) Threat sector

(2) Carrier at the origin ■

(3) Ship with coordinates (x_i, y_i)

Figure 3.1 Carrier, AAW Ship and Target

The carrier is positioned at the origin. The AAW ship has coordinates (x_i, y_i) which represent its location. We consider the scenario pictured in Figure 3.1.

3.2.1 OPTIMUM INTERCEPTION DISTANCE

We recall that we defined the optimum interception distance, the intercept distance, to be the shortest distance between ship i and the target trajectory. It is at this distance that the ship will try to kill the target. We showed in Section 2.2.1 that d^*_i was the intercept distance and that

$$d^*_i = x_i \sin\theta - y_i \cos\theta \tag{3.1}$$

where,

(x_i, y_i) are the coordinates of ship i, and

θ is the angle of the approaching AAW target.

3.2.2. THE EFFECT OF THE SHIP'S DISTANCE TO THE CARRIER

In Section 2.2.2.1 we established that the ships needed to maintain a certain distance from the carrier in order to detect the targets. We suggested incorporating this constraint into the model by introducing the function P_{rA_i}, for every ship i. We recall from Eq'n (2.4) that

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} r_i)} \tag{3.2}$$

where,

$r_i = \sqrt{x_i^2 + y_i^2}$ is the distance of ship i to the carrier, and

k_{A_i} is a scale factor.

3.2.3 PROBABILITY OF INTERCEPTION

We recall that each ship i has associated with it a function P_{A_i} that represents its ability to kill AAW targets. P_{A_i} was described fully in Section 2.2.2.2 , Eq'n (2.5) and is given by:

$$P_{A_i} = P_{r_{A_i}} \; e^{(-d_{*i}^2/\Sigma_{A_i})} \tag{3.3}$$

where,

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} r_i)}, \tag{3.2}$$

$$d_{*i} = (x_i \sin\theta - y_i \cos\theta), \tag{3.1}$$

Σ_{A_i} is the AAW range factor and,

θ is the angle of approach of the AAW target.

Figure 3.2 shows P_{A_i} .

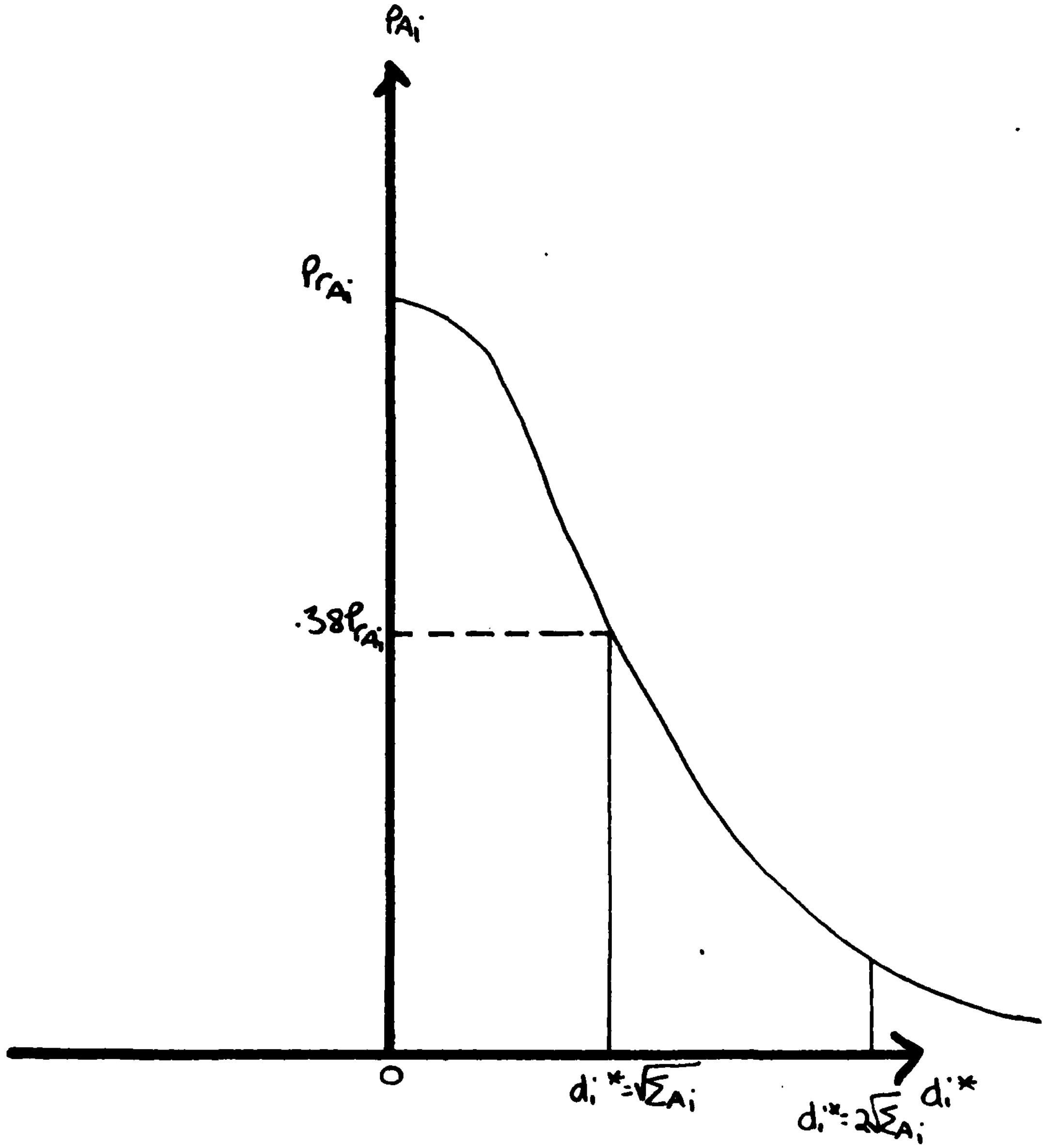


Figure 3.2 Probability of Kill, P_{A_i}

3.2.4 PROBABILITY DISTRIBUTION OF THE TARGET

The targets are uniformly distributed over a known sector. As in Section 2.2.2.3, let $f(\theta)$ be the probability density function of a single target. Then $f(\theta)$ is given by

$$f(\theta) = 1/(b - a) \qquad a \leq \theta \leq b$$
$$= 0 \qquad \text{otherwise}$$

3.2.5 SURVIVAL PROBABILITY OF THE CARRIER

We showed in Section 2.2.4.1 Case 1 that the survival probability of the carrier was given by

$$\text{Prob(carrier survives)} = \text{Prob(AAW ship kills the target over sector (a,b))}$$
$$= \int_a^b f(\theta) \times P_{A_i} \, d\theta \qquad (3.1)$$

Figure 3.3 shows the area of interest in the survival probability of the carrier.

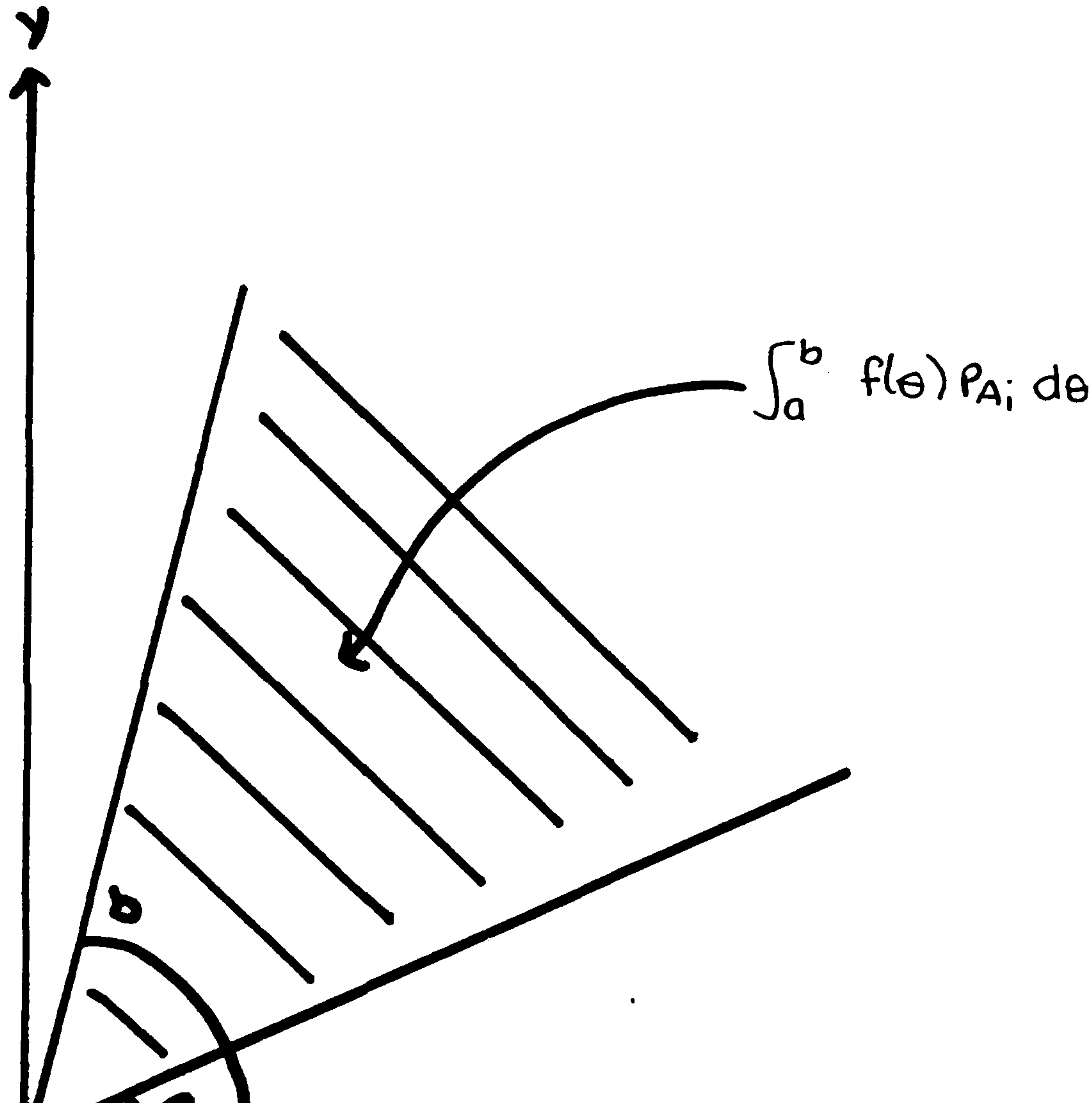


Figure 3.3 Area of Survival Probability

3.3 AN AAW SHIP - EXPERIMENTAL RESULTS

The model is complete. We will do a sensitivity analysis in order to better understand the properties of the model and its physical limitations. We will start by investigating the effect of different of k_{A_i} and Σ_{A_i} on the objective function and on the final position of the ship. We will examine the results of changing the size of the sector, study the effect of multiple targets, and explore the possibility of local maxima.

We will use the following notation throughout the entire chapter.

NOTATION

The maximum probability of the carrier survival = P_f

This is the optimal value of our objective function.

The optimal distance of the ship from the carrier is denoted R, and its optimal angle is ϕ. Thus, (R ,ϕ) denotes the optimal ship location in polar coordinates with respect to the carrier.

3.3.1 SUITABLE k_{A_i} AND Σ_{A_i} VALUES

We consider again Eq'n (3.3) which represents the AAW ship's ability to kill targets.

$$P_{A_i} = P_{r_{A_i}} \; e^{(-d*_i^2 / \Sigma_{A_i})} \qquad (3.3)$$

where,

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} r_i)} \qquad (3.2)$$

$$d*_i = x_i \sin\theta - y_i \cos\theta \qquad (3.1)$$

Σ_{A_i} is the AAW range factor, and

θ is the angle of approach of the target.

We want to maximize the probability of survival of the carrier given an AAW threat.

The optimization of this survival probability depends on the ability of a particular AAW ship to kill the target. Each AAW ship has associated with it a characteristic Σ_{A_i} value and k_{A_i} value. In this section, we state criteria for optimality (i.e. $P_f > .9$ and $R \cong 20$) and look for a "standard" AAW ship: one with Σ_{A_i} and k_{A_i} which satisfy these conditions for optimality.

What are suitable values for k_{A_i} and Σ_{A_i} ? In order to determine them, we ran some experiments on the computer.

EXPERIMENT 3.1

Purpose: To find values for k_{A_i} and Σ_{A_i} such that the final probability of survival of the carrier, P_f, is greater than .9 and R, the final position of the AAW ship is about 20. The sector extends from 0-45 and has a single target threat.

Table 3.10 shows the value of the fixed parameters in Experiment 3.1.
Table 3.11 shows the effect of setting $k_{A_i} = 0.2$ and varying Σ_{A_i} .
Table 3.12 shows the effect of setting Σ_{A_i} = 1000 and varying k_{A_i} .

DISCUSSION OF RESULTS (Table 3.11)

Consider Eq'n (3.4).

$$\text{Prob(carrier survives)} = \int_0^{45} f(\theta) \times P_{A_i} \, d\theta \qquad (3.4)$$

where,

Number of Ships	Type of Ship(s)	Number of Sectors	Type of Sectors	Sector	Number of Targets
1	AAW	1	AAW	0-45	1

Table 3.10. Fixed Parameters in Exp't 3.1: 1 Ship

Σ_{A_i}	1000	800	500	200	50
Surv. Prob. P_f	.964	.959	.944	.902	.794
Ship Range R	22.5	21.69	19.86	16.43	11.76
Ship Angle ϕ	22.5	22.5	22.5	22.5	22.5

Table 3.11 Effect of Varying Σ_{A_i} (k_{A_i} = 0.2)

Σ_{A_i}	.05	0.1	0.2	0.3	0.4
Surv. Prob. P_f	.815	.913	.964	.979	.986
Ship Range R	49.85	34.49	22.5	17.26	14.14
Ship Angle ϕ	22.5	22.5	22.5	22.5	22.5

Table 3.12 Effect of Varying k_{A_i} (Σ_{A_i} = 1000)

$$P_{A_i} = P_{r_{A_i}} \; e^{(-{d^*_i}^2 / \Sigma_{A_i})}$$

and,

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} r_i)} ,$$

$$d^*_i = x_i \sin\theta - y_i \cos\theta$$

It is evident that as Σ_{A_i} increases, the probability of survival increases. So we expect in Table 3.11, that a larger Σ_{A_i} value would yield a larger P_f . This seems to be the case. We would also suspect that a larger Σ_{A_i} would result in the distance of the ship from the carrier, R, to be larger because a bigger range means that a ship need not be so close to the carrier. Because of the symmetry of the problem, and the uniform distribution of the target within the threat sector, the final angle of the ship in all the examples is 22.5°.

DISCUSSION OF RESULTS (Table 3.12)

Again, from observing Eq'n (3.4), we see that as the value of k_{A_i} increases, the value of the probability of survival increases. This is consistent with P_f increasing with k_{A_i}.

3.3.2.2 A CHANGE IN SECTOR SIZE

We now increase the size of the sector from 0 - 90 and consider the next experiment.

EXPERIMENT 3.2

Purpose: To increase the sector size from 0 - 90 and, using the same values of k_{A_i} and Σ_{A_i} as in Experiment 3.1, compare the values of P_f and R with those of Experiment 3.1.

Table 3.20 shows the values of the fixed parameters in Experiment 3.2. Table 3.21 shows the effect of setting $k_{A_i} = 0.2$ and varying Σ_{A_i}. Table 3.22 shows the result of setting $\Sigma_{A_i} = 1000$ and varying k_{A_i}.

DISCUSSION OF RESULTS (Table 3.21 and Table 3.22)

As we expect, when the threat sector is expanded the ship is no longer as effective and the carrier survival probability value, P_f, drops. The optimal ship-to-carrier distance, R, decreases because the ship must be pulled toward the carrier in order to better cover the carrier. Once more, because of symmetry, the angle of the defending ship is 45°, at the center of the threat sector.

In the next experiment, we observe what happens when we change the size of the sector and leave all other parameters fixed.

Number of Ships	Type of Ship(s)	Number of Sectors	Type of Sector(s)	Sector	Number of Targets
1	AAW	1	AAW	0-90	1

Table 3.20 Fixed Parameters in Exp't 3.2: 1 Ship

Σ_{A_i}	1000	800	500	200	50
Surv. Prob. P_f	.918	.906	.877	.802	.644
Ship Range R	17.57	16.75	15.07	12.02	8.09
Ship Angle ϕ	45	45	45	45	45

Table 3.21 Effect of Varying Σ_{A_i} (k_{A_i} = 0.2)

Σ_{A_i}	.05	0.1	0.2	0.3	0.4
Surv. Prob. P_f	.672	.823	.918	.951	.966
Ship Range R	34.67	25.47	17.57	13.77	11.47
Ship Angle ϕ	45	45	45	45	45

Table 3.22 Effect of Varying k_{A_i} (Σ_{A_i} = 1000)

EXPERIMENT 3.3

Purpose: To observe the effect on the carrier survival probability, P_f,
and on the ship range, R, when we vary the size of the sector.

Table 3.30 shows the values of the fixed parameters in Experiment 3.3. Table 3.31 shows the effect of setting $\Sigma_{A_i} = 1000$ and $k_{A_i} = .2$ and varying the size of the sector.

DISCUSSION OF RESULTS (Table 3.31)

Again, we observe that as the size of the sector is increased, it becomes more difficult for the ship to protect the carrier and as a result, the optimal survival probability, P_f, decreases. Also, as the size of of the sector increases, the optimal ship-to-carrier distance, R, shrinks because the ship needs to move closer to the carrier in order to better defend it.

3.3.2.3 MULTIPLE TARGETS

What happens to the probability of survival and the final location of the ship when the number of targets in a sector is increased to N?

The survival probability of the carrier based on this scenario is:

$$\text{Prob(carrier survives)} = \left\{ \int_a^b f(\theta) \times P_{A_i} \, d\theta \right\}^N \tag{3.5}$$

Number of Ships	Type of Ship(s)	Number of Sectors	Type of Sectors	$\sum A_i$	k_{A_i}	Number of Targets
1	AAW	1	AAW	1000	0.2	1

Table 3.30 Fixed Parameters in Exp't 3.3: 1 Ship

Sector	0-15	0-20	0-45	0-60	0-75	0-90	0-105	0-120
P_f	.992	.988	.964	.948	.933	.918	.904	.891
R	31.63	29.18	22.5	20.40	18.80	17.57	16.59	15.81
ϕ	7.5	10	22.5	30	37.5	45	52.5	60

Table 3.31 Effect of Varying the Sector Size

We expect P_f to decrease since the threat is more dense. Since the threats are independent and identically distributed, we expect that R, the optimal ship-to-carrier distance, will not change.

CLAIM 3.1:

For independently and identically distributed targets, R does not change with the number of targets.

Proof

Please refer to Appendix A for the proof.

We now verify this with experimental results.

EXPERIMENT 3.4

Purpose: To see the effect of multiple targets on P_f and R.

Table 3.40 shows the values of the fixed parameters in Experiment 3.4.

Table 3.41 shows the result of varying the number of targets.

Number of Ships	Type of Ship(s)	Number of Sectors	Type of Sector	Sector	Σ_{A_i}	k_{A_i}
1	AAW	1	AAW	0-45	1000	0.2

Table 3.40 Fixed Parameters in Exp't 3.4: 1 Ship

Number of Targets	1	2	5	10	20
Surv. Prob. P_f	.964	.930	.834	.696	.485
Ship Range R	22.58	22.58	22.58	22.58	22.58
Ship Angle ϕ	22.5	22.5	22.5	22.5	22.5

Table 3.41 Effect of Varying the Number of Targets

DISCUSSION OF RESULTS (Table 3.41)

We see that the ship-to-target distance, R, remains constant (as does the optimal angle of the ship) as the number of targets in the sector changes. This is a direct result of CLAIM 3.1 above.

We also observe that the carrier survival probability, P_f , decreases rapidly as the number of targets increases substantially. When we double the sector size to 0-90, and set $\Sigma_{A_i} = 1000$, $k_{A_i} = 0.2$, then $P_f = .918$ (Table 3.21). In Experiment 3.3, Table 3.31, with $\Sigma_{A_i} = 1000$, $k_{A_i} = 0.2$ and a 0-45 sector but with double the concentration of targets, $P_f = .930$. Comparing these two results, it is more detrimental in this case to double the sector size than to double the concentration of targets.

3.3.3 EXISTENCE OF LOCAL MAXIMA

In this section, we explore the possibility of local maxima. That is, we vary the initial position of the AAW ship to determine whether it has any influence on the optimal position of the ship. If the optimal location of the ship is independent of the initial positions,we conclude that there are no local maxima. We make intelligent choices for the initial conditions. That is, we choose a position which seems "reasonable". If we want the optimal ship-to-carrier distance, R, to be about 20, we would not start with an ini-

tial position of the ship at (80,80). Not only may it be costly because of the incremental number of iterations that might be required to converge to the (desired) optimal solution, but also such choices increase the probability of falling on a local maximum, which yields an undesirable (i.e. too far away) location. In some cases, choosing an initial condition which is "far" from the carrier (such as (80,80) in Table 3.52) results in the same P_f but shifts the optimal angle of the ship by 180 degrees. These are limitations of the model.

We recall that we are dealing with permissible sectors. A permissible sector was defined (Def'n 2.8) to be one in which, for a given ship with coordinates $(x_i^{\cdot}, y_i^{\cdot})$, d_i^*, the intercept distance, is given by:

$$d_i^* = x_i^{\cdot}\sin\theta - y_i^{\cdot}\cos\theta \qquad \text{for all } \theta,\ a \leq \theta \leq b$$

(See Section 2.2.3.1, Figure 2.12b for an example of a position (or initial condition) of a ship which does not generate a permissible sector.)

The initial condition that we select for our ship must generate a permissible sector.

EXPERIMENT 3.5

Purpose: To test for the existence of local maxima. We keep all other parameters fixed and vary the initial position of the ship.

Table 3.50 shows the values of the fixed parameters in Experiment 3.5. Table 3.51 shows the result of changing the initial position of the ship.

DISCUSSION OF RESULTS (Table 3.51)

We observe that for "reasonable" initial conditions (i.e. ones that generate permissible sectors and are not too far from the desired optimal ship-to-carrier distance, R, the model appears not to be sensitive to changes in the initial condition. In our examples, at worst the optimal angle of the ship shifted by 180 degrees and this presented an unrealistic solution for our problem. The optimal survival probability, P_f, did not change.

3.4 AN ASW SHIP - EXPERIMENTAL RESULTS

In this section, we perform experiments on an ASW ship similar to those that we performed on the AAW ship in Section 3.3. The purpose is to discover any trends in the optimal carrier survival probability, P_f , and in the optimal position, R, of the ASW ship.

Number of Ships	Type of Ship(s)	Number of Sectors	Type of Sector	Sector	Σ_{A_i}	k_{A_i}	Number of Targets
1	AAW	1	AAW	0-45	1000	0.2	1

Table 3.50 Fixed Parameters in Exp't 3.5: 1 Ship

Init. Cond'n	(5,1)	(20,20)	(1,30)	(80,80)	(40,0)	(0,3)	(1,-1)
Surv. Prob. P_f	.964	.964	.964	.964	.964	.964	.964
Ship Range R	22.58	22.58	22.58	22.58	22.58	22.58	22.58
Ship Angle ϕ	22.5	22.5	22.5	202.5	22.5	22.5	22.5

Table 3.51 Effect of Changing the Initial Condition

We will discuss the results of changing the size of the sector, study the effect of multiple targets, and review the possibility of local maxima.

The reader should be reminded of the fact that in our model the motion and interception of submarines is treated in a similar manner as that of an air threat. Each submarine target is assumed to follow a straight line path to the carrier. This assumption implies that we are treating a submarine threat as though it has to closely approach the carrier before it launches a torpedo; it does not attempt to evade ASW weapons. Also, the fact that submarines can create air threats, by launching missiles against the carrier, is not addressed (unless separately modeled in the AAW problem) in our model* .

We emphasize that this "symmetry" of AAW and ASW threats is a major limitation in the practical application of our model. Further research is needed to improve the relevance of the ASW part of our model.

3.4.1 SUITABLE K_{si} AND Σ_{si} VALUES

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* The model developed by Castanon et al (3) does treat this problem.

We consider an analogous equation to Eq'n (3.3) - the ASW ship's ability to kill ASW targets.

$$P_{S_i} = P_{r_{S_i}} \; e^{(-d_i^{*2} / \Sigma_{S_i})} \tag{3.5}$$

where,

$$P_{r_{S_i}} = 1 - e^{-(k_{S_i} \, r_i)}$$

$$d^*_i = x_i \sin\theta \; - \; y_i \cos\theta$$

Σ_{S_i} is the ASW range factor, and

θ is the angle of approach of the ASW target.

Our purpose is to maximize the probability of survival of the carrier, given a single ASW target threat. The optimization of this survival probability depends on the ability of a particular ASW ship to kill the target. Each ASW ship has associated with it a characteristic Σ_{S_i} and k_{S_i} value. We impose criteria for optimality: the value of P_f must exceed .9 and the optimal ship-to-carrier distance R, must be approximately 50.

We look for an ASW ship that satisfies these conditions, i.e. we want to determine the characteristic k_{S_i} and Σ_{S_i} values of an ASW ship.

What are suitable values for k_{S_i} and Σ_{S_i} ? We run the next experiment to determine them.

EXPERIMENT 3.6

Purpose: To find values for k_{S_i} and Σ_{S_i} such that the final probability of survival of the carrier, P_f, is greater than .9 and R, the final position of the AAW ship is about 50.
The sector extends from 0-45 and has a single target threat.

Table 3.60 shows the value of the fixed parameters in Experiment 3.6.
Table 3.61 shows the effect of setting $k_{S_i} = 0.07$ and varying Σ_{S_i} .
Table 3.62 shows the effect of setting Σ_{S_i} = 2000 and varying k_{S_i} .
Table 3.63 shows the effect of setting Σ_{S_i} = 8000 and varying k_{S_i} .

DISCUSSION OF RESULTS (Table 3.61)

Consider Eq'n (3.6).

$$\text{Prob(carrier survives)} = \int_0^{45} f(\theta) \times P_{S_i} \, d\theta \qquad (3.6)$$

where,

Number of Ships	Type of Ship	Number of Sectors	Type of Sector	Sector	Number of Targets
1	ASW	1	ASW	0-45	1

Table 3.60 Fixed Parameters in Exp't 3.6: 1 Ship

Σ_{S_i}	8000	5000	3000	2600	2200	2000	1000
P_f	.964	.950	.931	.925	.917	.912	.869
R	64.30	58.99	53.38	51.84	50.06	49.05	42.00
ϕ	22.5	22.5	22.5	22.5	22.5	22.5	22.5

Table 3.61 Effect of Varying Σ_{S_i} (k_{S_i} = .07)

k_{s_i}	.02	.05	.07	.08	.09	.10
P_f	.692	.871	.912	.925	.935	.943
R	91.14	59.04	49.05	45.40	42.33	39.72
ϕ	22.5	22.5	22.5	22.5	22.5	22.5

Table 3.62 Effect of Varying k_{s_i} (Σ_{s_i} = 2000)

k_{s_i}	.02	.05	.07	.08	.09	.10
P_f	.837	.943	.964	.969	.974	.978
R	132.63	79.43	64.30	58.95	54.53	50.81
ϕ	22.5	22.5	22.5	22.5	22.5	22.5

Table 3.63 Effect of Varying k_{s_i} (Σ_{s_i} = 8000)

$$P_{S_i} = P_{r_{S_i}} \; e^{(-d*_i^2 / \Sigma_{S_i})}$$

$$P_{r_{S_i}} = 1 - e^{-(k_{S_i} \, r_i)} ,$$

and,

$$d*_i = x_i \sin\theta \; - \; y_i \cos\theta$$

It is evident that as Σ_{S_i} increases, the probability of survival increases. So we expect in Table 3.61, that a larger Σ_{S_i} value would yield a larger P_f . This seems to be true. We would also expect that a larger Σ_{S_i} would result in the distance of the ship from the carrier, R, to be larger because a bigger range means that a ship need not be so close to the carrier. Because of the symmetry of the problem, and the uniform distribution of the target, the final angle of the ship in all the examples is 22.5°.

DISCUSSION OF RESULTS (Tables 3.62 and 3.63)

Again, from observing Eq'n (3.6), we see that as the value of k_{S_i} increases, the value of the probability of survival increases. This is consistent with P_f increasing with k_{S_i}. A larger Σ_{S_i} increases the P_f value and also increases the value of R.

3.3.2.2 A CHANGE IN SECTOR SIZE

We now increase the size of the sector from 0 - 90 and consider the next experiment.

EXPERIMENT 3.7

Purpose: To increase the sector size from 0 - 90 and, using the same values of k_{S_i} and Σ_{S_i} as in Experiment 3.6, compare the values of P_f and R with those of Experiment 3.6.

Table 3.70 shows the values of the fixed parameters in Experiment 3.7. Table 3.71 shows the effect of setting k_{S_i} = 0.07 and varying Σ_{S_i}. Table 3.72 shows the result of setting Σ_{S_i} = 2000 and varying k_{S_i}.

DISCUSSION OF RESULTS (Table 3.71 and Table 3.72)

As we expect, when the threat sector size is increased the ship can no longer defend the carrier as well, and the value of P_f decreases. Also, the optimal ship-to-carrier distance, R, decreases since the ship must get closer to the carrier in order to better defend it.

Number of Ships	Type of Ship	Number of Sectors	Type of Sector	Sector	Number of Targets
1	ASW	1	ASW	0-90	1

Table 3.70 Fixed Parameters in Exp't 3.7: 1 Ship

Σ_{S_i}	8000	5000	3000	2600	2200	2000	1000
P_f	.917	.890	.855	.843	.829	.821	.752
R	49.99	45.11	40.03	38.66	37.08	36.20	30.09
ϕ	45	45	45	45	45	45	45

Table 3.71 Effect of Varying Σ_{S_i} (k_{S_i} = .07)

k_{S_i}	.02	.05	.07	.08	.09	.10
P_f	.522	.754	.821	.844	.862	.877
R	60.22	42.30	36.20	33.87	31.87	30.14
ϕ	45	45	45	45	45	45

Table 3.72 Effect of Varying k_{S_i} (Σ_{S_i} = 2000)

3.4.2.3 MULTIPLE TARGETS

What happens to the probability of survival and the final location of the ship when the number of targets in a sector is increased to Q ?

The survival probability of the carrier is:

$$\text{Prob(carrier survives)} = \left\{ \int_{0}^{45} f(\theta) \times P_{S_i} \, d\theta \right\}^{Q} \qquad (3.5)$$

The situation is completely analogous to the one in Section 3.3.2.3 for AAW ships, and we expect the same results. We expect P_f to decrease since the threat is more dense. Since the threats are independent and identically distributed, we expect that R, the optimal ship-to-carrier distance, will not change.

We present the next experiment to support this hypothesis.

EXPERIMENT 3.8

Purpose: To see the effect of multiple targets on P_f and R.

Table 3.80 shows the values of the fixed parameters in Experiment 3.8. Table 3.81 shows the result of varying the number of targets.

DISCUSSION OF RESULTS (Table 3.81)

For a maximum of twelve targets, the program performs well. The value of P_f dropped as the number of targets increased, as we expected. The position of the ship was not expected to change, (based on CLAIM 3.1 of Section 3.3.2.3), and it did not. Why can the program not work for a number of targets greater than fifteen? There is no relative function convergence, only convergence of the iterates, (x_i, y_i), and this occurs in two iterations. This has no physical importance. Because we are solving this problem numerically, using a standard optimization routine, we are at the mercy of its limitations. The result is that we must put an upper limit of fifteen on the number of targets. However, such limitations occur only when the carrier survival probability is almost zero; hence, such shortcomings have little impact on more realistic scenarios.

The following experiment shows what happens with different values of k_{S_i} and Σ_{S_i}.

Number of Ships	Type of Ship	Number of Sectors	Type of Sector	Sector	Σ_{si}	k_{si}
1	ASW	1	ASW	0-45	2000	.07

Table 3.80 Fixed Parameters in Exp't 3.8: 1 Ship

Number of Targets	1	2	5	10	12	14	15	20
P_f	.912	.832	.633	.401	.334	.3.01	$.1 \times 10^{-7}$	$.4 \times 10^{-10}$
R	49.05	49.05	49.05	49.05	49.05	49.05	5.09	5.09
ϕ	22.5	22.5	22.5	22.5	22.5	22.5	11.31	11.31

Table 3.81 Effect of Varying the Number of Targets

EXPERIMENT 3.9

Purpose: To see the effect of multiple targets on P_f and R when Σ_{S_i} =8000 and k_{S_i} =.1.

Table 3.90 shows the values of the fixed parameters in Experiment 3.9. Table 3.91 shows the result of varying the number of targets.

DISCUSSION OF RESULTS (Table 3.91)

As the number of targets increased from one to eighteen, the value of P_f decreased, and the position of the ship did not change. This we expected since the ship is no longer as effective, and the targets are independent of each other. When the number of targets was set to nineteen, the program failed. As in Experiment 3.8, there was no relative function convergence, only convergence of the iterates, which is meaningless. It is worthwhile to note that for Σ_{S_i} = 2000, k_{S_i} = .07, (Exp't 3.8) the program could sustain only fourteen targets. In Experiment 3.9, with Σ_{S_i} = 8000, k_{S_i} = .1, the program worked for nineteen targets. Also, in Experiment 3.4 of the AAW ship, the program worked for twenty targets.

3.4.3 EXISTENCE OF LOCAL MAXIMA

In Section 3.3.3, we considered the existence of local maxima. That is, we observed the effect of the initial position of the AAW ship on its optimal position. In this section, we do the same. Again we make intelligent choices for the initial conditions for the same reasons presented in Section 3.3.3.

Number of Ships	Type of Ship	Number of Sectors	Type of Sector	Sector	Σ_{si}	k_{s_1}
1	ASW	1	ASW	0-45	8000	.10

Table 3.90 Fixed Parameters in Exp't 3.9: 1 Ship

Number of Targets	1	2	5	10	15	18	19	20
P_ϕ	.978	.956	.894	.800	.716	.670	$.3 \times 10^{-7}$	$.1 \times 10^{-10}$
R	50.81	50.81	50.81	50.81	50.81	50.81	5.09	5.09
ϕ	22.5	22.5	22.5	22.5	22.5	22.5	11.3	11.3

Table 3.91 Effect of Varying the Number of Targets

EXPERIMENT 3.10

Purpose: To determine if there are any local maxima.

Table 3.100 shows the values of the fixed parameters in Experiment 3.10.

Table 3.101 shows the effect on P_f and R of changing the initial condition.

DISCUSSION OF RESULTS (Table 3.101)

We observe that for "reasonable" initial conditions, the model appears to be robust in the sense that the optimal survival probability, P_f, and the optimal ship-to-carrier distance, R , are unaffected.

3.5 PROBLEM STATEMENT - SINGLE SHIP WITH DUAL CAPABILITIES

The purpose of the problem is to maximize the probability of survival of a carrier given a single air and a single sub threat. The carrier is being defended by either a single AAW ship or a single ASW ship. The AAW and ASW threat sectors are identical. The solution to the problem is the optimum location of the AAW (ASW) ship with respect to the carrier; that position which maximizes the survival probability of the carrier. The problem is formulated in terms of an AAW ship; the formulation for an ASW ship is analogous.

Number of Ships	Type of Ship	Number of Sectors	Type of Sector	Sector	Σ_{s_i}	k_{s_i}	Number of Targets
1	ASW	1	ASW	0-45	8000	.10	1

Table 3.100 Fixed Parameters in Exp't 3.10: 1 Ship

Init. Cond'n	(0,3)	(30,0)	(80,80)	(1,-1)
Surv. Prob. P_f	.978	.978	.978	.978
Ship Range R	50.81	50.81	50.81	50.81
Ship Angle ϕ	22.5	22.5	22.5	22.5

Table 3.101 Effect of Changing the Initial Condition

3.6 MATHEMATICAL FORMULATION - DUAL CAPABILITIES

The carrier is located at the origin. The AAW ship has coordinates (x_i, y_i) which represent its position. There are two identical threat sectors, (a,b), one AAW, one ASW. The scenario is pictured in Figure 3.4.

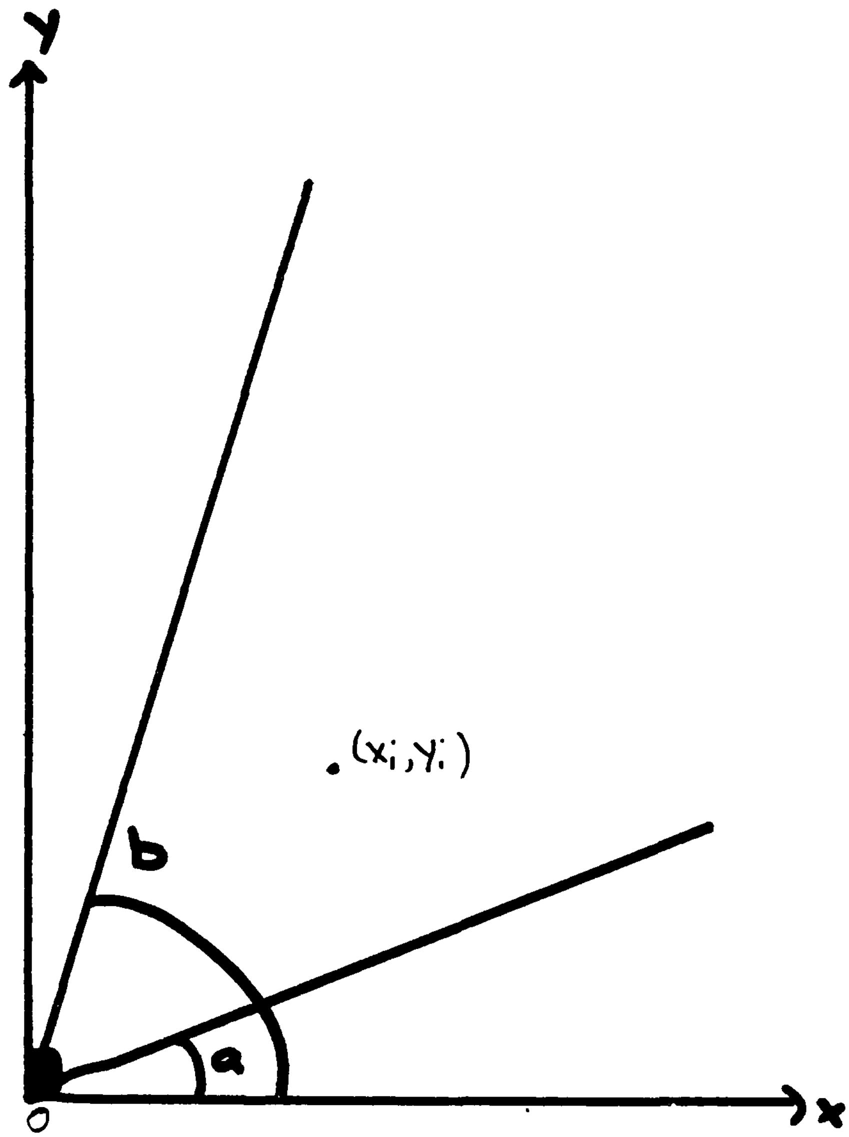
3.6.1 OPTIMUM INTERCEPTION DISTANCE

The intercept distance is the shortest distance between ship i and the target trajectory. In this case, it is the shortest distance between ship i and either an AAW or ASW target. From Section 2.2.1, d_i^* is the intercept distance and

$$d_i^* = x_i \sin\theta - y_i \cos\theta \qquad \text{for all } \theta,\ a \leq \theta \leq b.$$

3.6.2 THE EFFECT OF THE SHIP'S DISTANCE TO THE CARRIER

As in Section 2.2.2.1, we have a function that reflects the ships' need to maintain a certain distance from the carrier. We recall from Eq'n (3.4) that



<u>Legend</u>

1) Ship i with Coordinates (x_i, y_i)

2) Carrier at the Origin ■

3) Threat Sector (a ,b): Identical for AAW and ASW

Figure 3.4 Single Ship, Dual Capabilities

$$P_{rA_i} = 1 - e^{-(k_{A_i} r_i)} \tag{3.4}$$

where,

$r_i = \sqrt{x_i^2 + y_i^2}$ is the distance of ship i to the carrier, and

k_{A_i} is a parameter.

In the case of an AAW ship with dual capabilities, it is no longer sufficient to represent the effect of the ship-to-carrier distance by P_{rA_i} of Eq'n (3.4) This is so because an additional threat (the ASW one) of a different nature has been introduced. The optimal position of the AAW ship will depend on both its AAW and ASW defense capabilities. The latter are captured independently in the optimal ship-to-carrier distance, R. Consequently, we introduce an analogous function to that of Eq'n (3.4), and say that the effect of the AAW ship's distance to the carrier is described completely by both of these functions.

This new function is:

$$P_{r_{S_i}} = 1 - e^{-(k_{S_i}\ r_i)}\ ,\ k_i > 0 \tag{3.6}$$

where,

$r_i = \sqrt{x_i^2 + y_i^2}$ is the distance of ship i to the carrier, and

k_{S_i} is a parameter.

3.6.3 PROBABILITY OF INTERCEPTION

The ability of a ship to kill AAW targets is given by

$$P_{A_i} = P_{r_{A_i}}\ e^{(-d_i^{*2} / \Sigma_{A_i})} \tag{3.3}$$

where,

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i}\ r_i)}\ , \tag{3.2}$$

$$d_i^* = x_i \sin\theta - y_i \cos\theta\ , \tag{3.1}$$

Σ_{A_i} is the AAW range factor and

θ is the angle of approach of the AAW target.

Similarly, we said that the ability of a ship to kill ASW targets was given by an analogous function, P_{S_i} , where

$$P_{S_i} = P_{r_{S_i}} \; e^{(-d_i^{*2} / \Sigma_{S_i})} \tag{3.7}$$

where,

$$P_{r_{S_i}} = 1 - e^{-(k_{S_i} \, r_i)}, \quad k_{S_i} > 0$$

$d_i^* = x_i \sin\theta \; - \; y_i \cos\theta$,

Σ_{S_i} the ASW range factor and,

θ is the angle of approach of the ASW target.

3.6.4 PROBABILITY DISTRIBUTIONS OF THE TARGETS

Both the AAW and the ASW targets are uniformly distributed over the same (known) sector. Let $f(\theta)$ be the probability density function of a single AAW target. Then $f(\theta)$ is given by

$$f(\theta) = 1/(b - a) \qquad a \leqslant \theta \leqslant b$$
$$= 0 \qquad \text{otherwise}$$

If $g(\theta)$ is the probability density function of a single ASW target, then $g(\theta) = f(\theta)$, since they are distributed identically.

3.6.5 SURVIVAL PROBABILITY OF THE CARRIER

The probability that the carrier survives is the probability that the AAW ship kills the AAW target and the ASW target. This is given by

$$\begin{aligned}\text{Prob(carrier survives)} &= \text{Prob(AAW ship kills the AAW and the ASW target in sector (a,b))} \\ &= \int_a^b f(\theta)\, P_{A_i}\, d\theta \cdot \int_a^b f(\theta)\, P_{S_i}\, d\theta \qquad (3.8)\end{aligned}$$

3.7 AN AAW SHIP - EXPERIMENTAL RESULTS (DUAL CAPABILITIES)

We have built a model that captures the dual capabilities of an AAW ship, and the survival probability of the carrier under an AAW and an ASW threat. Our goal is to maximize the probability of survival of the carrier given an AAW and an ASW threat. The optimization of this survival probability depends on the AAW ship's ability to kill the AAW and the ASW target. We impose criteria for optimality: the value of the carrier survival probability, P_f, must exceed .80, and the the optimal ship-to-target distance, R, must be approximately 20 miles. In Experiment 3.1 we imposed similar conditions: we said that P_f had to exceed .9 and that R should be about 20 miles. In that experiment, the AAW ship had only AAW capabilities and was defending the carrier against a single AAW target. In this section, we consider the same ship but with both AAW and ASW capabilities, required to defend the carrier against both an AAW and an ASW target. We do not expect the ship to perform as well

because of the dual threat. We require it, however, to remain about 20 miles from the carrier.

We wish to mathematically describe the AAW ship that satisfies these criteria for optimality by finding the numerical values for k_{A_i}, k_{S_i}, Σ_{A_i} and Σ_{S_i}.

3.7.1 SUITABLE K_{A_i}, K_{S_i}, Σ_{A_i}, AND Σ_{S_i} VALUES

We consider Eq'ns (3.3) and (3.7) which represent the AAW ship's ability to kill AAW and ASW targets respectively.

$$P_{A_i} = P_{r_{A_i}} \quad e^{(-d*_i^2 / \Sigma_{A_i})} \tag{3.3}$$

where,

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} r_i)}, \quad k_{A_i} > 0$$

$$d*_i = x_i \sin\theta - y_i \cos\theta$$

Σ_{A_i} the AAW range factor and,

θ is the angle of approach of the AAW target.

Also,

$$P_{S_i} = P_{r_{S_i}} \, e^{(-d_i^{*2} / \Sigma_{S_i})} , \qquad (3.7)$$

where,

$$P_{r_{S_i}} = 1 - e^{-(k_{S_i} r_i)} , \quad k_{S_i} > 0$$

$d_i^* = x_i \sin\theta - y_i \cos\theta$

Σ_{S_i} is the ASW range factor and

θ is the angle of approach of the ASW target.

What are suitable values for k_{A_i}, k_{S_i}, Σ_{A_i} and Σ_{S_i} ?

EXPERIMENT 3.11

Purpose: To find values for k_{A_i}, k_{S_i}, Σ_{A_i}, and Σ_{S_i}, such that the probability of survival of the carrier, P_{ρ}, is greater than .8 and R, the optimal position of the AAW ship is about 20. The AAW and ASW sectors extend from 0-45 and both have a single target threat.

Table 3.110 shows the value of the fixed parameters in Experiment 3.11. Table 3.111 shows the effect of setting Σ_{S_i} = 500 and varying k_{S_i}.

Number of Ships	Type of Ship	Number of Sectors	Type of Sectors	Sectors	Number of Targets	Σ_{A_i}	k_{A_i}
1	AAW	2	1 AAW	0-45	1 AAW	1000	.2
			1 ASW	0-45	1 ASW		

Table 3.110 Fixed Parameters in Exp't 3.11: 1 Ship, Dual Capabilities

Σ_{S_i}	500	500	500	500
k_{S_i}	.02	.07	.09	.1
Surv. Prob. P_f	.448	.769	.817	.834
Ship Range R	48.13	31.70	28.57	27.34
Ship Angle θ	22.5	22.5	22.5	22.5

Table 3.111 Effect of Varying k_{S_i} (Σ_{S_i} = 500)

DISCUSSION OF RESULTS (Table 3.111)

In experimenting to find a "good" combination of Σ_{S_i} and k_{S_i} for the AAW ship one thing is clear. $\Sigma_{S_i} < \Sigma_{A_i}$ since the AAW ship has superior AAW capabilities. This implies that we are considering a subset of Σ_{S_i} , such that $\Sigma_{S_i} <$ Σ_{A_i} . In Experiment 3.1 of the single ship, single sector case, we found that $k_{A_i} = .2$ and $\Sigma_{A_i} = 1000$ yielded $P_f = .934$ and $R = 22.58$; reasonable results. In Table 3.11.1, the optimal ship- to-carrier distance, R, increased by about 5 miles. This is expected and acceptable since the ASW threat encourages a ship (AAW or ASW) to be even further from the carrier for ASW surveillance reasons. This is a result of the k_{S_i} value selected. The P_f value drops because the ship is faced with a dual threat.

Based on the results of Tables 3.111 we choose as our canonical values for the AAW ship:

$$k_{A_i} = .2$$

$$k_{S_i} = .1$$

$$\Sigma_{A_i} = 1000$$

and,

$$\Sigma_{S_i} = 500$$

3.8 AN ASW SHIP - EXPERIMENTAL RESULTS (DUAL CAPABILITIES)

As in Section 3.7, we wish to maximize the probability of survival of the carrier, under an AAW and an ASW threat. Now, however, the carrier is being defended by an ASW ship. We impose criteria for optimality: the optimal probability of survival, P_c must exceed .8 and optimal ship-to-carrier distance, R, must be approximately 50. We wish to characterize an ASW ship by finding canonical values for k_{A_i}, k_{S_i}, Σ_{A_i}, and Σ_{S_i} that satisfy these criteria.

3.8.1 SUITABLE K_{A_i}, K_{S_i}, Σ_{A_i} AND Σ_{S_i} VALUES

We consider Eq'ns (3.3) and (3.7) which represent the ASW ship's ability to kill AAW and ASW targets respectively.

$$P_{A_i} = P_{r_{A_i}} \quad e^{(-d*_i^{\,2} / \Sigma_{A_i})} \tag{3.3}$$

where,

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} r_i)} \quad , \; k_{A_i} > 0$$

$$d*_i = x_i \sin\theta \; - \; y_i \cos\theta$$

Σ_{A_i} is the AAW range factor and,

θ is the angle of approach of the AAW target.

Also,

$$P_{S_i} = P_{r_{S_i}} \; e^{(-d_i^{*2} / \Sigma_{S_i})} \tag{3.7}$$

where,

$$P_{r_{S_i}} = 1 - e^{-(k_{S_i} r_i)} \quad , \; k_{S_i} > 0$$

$$d_i^* = x_i \sin\theta - y_i \cos\theta$$

Σ_{S_i} is the ASW range factor and,

θ is the angle of approach of the ASW target.

What are suitable values for k_{A_i}, k_{S_i}, Σ_{A_i} and Σ_{S_i} ?

EXPERIMENT 3.12

Purpose: To find values for k_{A_i}, k_{S_i}, Σ_{A_i}, and Σ_{S_i}, such that the final probability of survival of the carrier, P_f, is greater than .8 and R, the optimal ship-to-carrier distance is about 50.
The AAW and ASW sectors extend from 0-45 and both have a single target threat.

Table 3.120 shows the values of the fixed parameters in Experiment 3.12.
Table 3.121 shows the effect of varying k_{A_i} .

Number of Ships	Type of Ship	Number of Sectors	Type of Sectors	Sectors	Number of Targets	Σ_{S_i}	k_{S_i}
1	ASW	2	1 AAW	0-45	1 AAW	8000	.10
			1 ASW	0-45	1 ASW		

Table 3.120 Fixed Parameters in Exp't 3.12: 1 Ship, Dual Capabilities

Σ_{A_i}	800	800	850
k_{A_i}	.05	.06	.06
Surv. Prob. P	.776	.807	.813
Ship Range R	47.75	44.63	45.17
Ship Angle ϕ	22.5	22	22.5

Table 3.121 Effect of Varying Σ_{A_i} and k_{A_i}

DISCUSSION OF RESULTS (Table 3.121)

We know that as k_{S_i} increases, P_f increases. Also as Σ_{S_i} increases P_f increases. By introducing an AAW threat, we expect the optimal ship-to-carrier distance, R, to decrease. Most importantly, we are restricted to a subset of Σ_{A_i} values. We know that Σ_{A_i} must be less than 8000 since an ASW ship has superior ASW capabilities. Also, Σ_{A_i} must be less than $1000 = \Sigma_{A_i}$ of the AAW ship. If not, then why bother having AAW ships if the ASW ships can be more effective against AAW targets?

Base on the results of Table 3.121 we select as our canonical values for the ASW ship:

$k_{A_i} = .2$

$k_{S_i} = .1$

$\Sigma_{A_i} = 850$

and,

$\Sigma_{S_i} = 8000$

We now have established characteristic k_{A_i}, k_{S_i}, Σ_{A_i}, and Σ_{S_i} values for both our AAW and ASW ships. These canonical ships will be used in Chapter 4 to investigate the behaviour of the model under different scenarios when two or more ships are protecting the carrier.

3.9 CONCLUDING REMARKS

In this chapter, we performed several numerical experiments. The purpose of these experiments was to observe the effect on the carrier survival probability and on the optimum location of the single AAW (or ASW) ship, under different scenarios.

CHAPTER 4 MULTIPLE SHIP CASE STUDIES WITH AAW AND ASW THREAT SECTORS

4.0 SUMMARY

We venture into the realm of two ships. We investigate how the two ships work in harmony, in order to realize the maximum survival probability of the carrier. We demand that the model trek into hazardous territory; pure threats, multiple targets, changes in the sector, tests for symmetry. We further impose the additional burden of three ships. But the model successfully glides through these tests, and emerges undaunted.

4.1 PROBLEM STATEMENT

The purpose of the problem is to maximize the probability of survival of a carrier given air and/or sub threats. The threats are uniformly distributed over known sectors. An AAW and an ASW ship are defending the carrier. The solution to the problem is the optimum locations of the AAW and the ASW ships with respect to the carrier; those locations that maximize the probability of survival of the carrier.

The purpose of this chapter is to study various scenarios of the two and three ship problems in order to see how the ships interact to best protect the carrier.

4.2 MATHEMATICAL FORMULATION: TWO SHIPS

The carrier is positioned at the origin. The AAW ship and the ASW ship have coordinates (x_1, y_1) and (x_2, y_2) respectively, which represent their locations relative to the carrier. In polar coordinates (R_1, ϕ_1) denotes the optimal location of the AAW ship, while (R_2, ϕ_2) denotes the optimal location of the ASW ship.

4.2.1 OPTIMUM INTERCEPTION DISTANCE

We defined the (optimum) interception distance to be the shortest distance between ship i and the target trajectory. This is the distance at which the ship attempts to kill the target. In Section 2.2.1, we said that d^*_i was the intercept distance and that

$$d^*_i = x_i \sin\theta - y_i \cos\theta \qquad (4.1)$$

where,

(x_i, y_i) are the coordinates of ship i, and

θ is the angle of approach of the target.

4.2.2 THE EFFECT OF THE SHIPS' DISTANCE TO THE CARRIER

In Section 3.6.2 we determined that the ships were required to keep a certain distance from the carrier. We introduced the functions $P_{r_{A_i}}$, and $P_{r_{S_i}}$, for every ship i, to capture this constraint. From Eq'ns (3.4) and (3.6) we have that

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} r_i)} , \; k_{A_i} > 0 \tag{4.2}$$

where,

$r_i = \sqrt{x_i^2 + y_i^2}$ is the distance of ship i to the carrier, and

k_{A_i} is a parameter associated with each cruiser and destroyer.

We also have

$$P_{r_{S_i}} = 1 - e^{-(k_{S_i} \; r_i \;)} \;, \quad k_{S_i} > 0 \tag{4.3}$$

where,

$r_i = \sqrt{x_i^2 + y_i^2}$ is the distance of ship i to the carrier, and

k_{S_i} is a parameter.

4.2.3 PROBABILITY OF INTERCEPTION

As we explained in Section 2.2.2.2, each ship has two functions which fully describe its ability to intercept targets. P_{A_i} of Eq'n(4.4) reflects ship i's ability to intercept an AAW target.

P_{A_i}, the PROBABILITY(SHIP I KILLS AN AAW TARGET), is

$$P_{A_i} = P_{r_{A_i}} \quad e^{(-d_i^{*2} / \Sigma_{A_i})} \tag{4.4}$$

where,

$$P_{r_{A_i}} = 1 - e^{-(k_{A_i} \; r_i \;)} \;, \quad k_{A_i} > 0 \tag{4.2}$$

$$d_i^* = (x_i \sin\theta - y_i \cos\theta), \tag{4.1}$$

Σ_{A_i} is the AAW range factor, and

θ is the angle of approach of the AAW target.

Similarly, for every ship i we have a function, P_{S_i}, which completely describes its ability to intercept ASW targets.

$$P_{S_i} = P_{r_{S_i}} \; e^{(-d{*}_i^{2} / \Sigma_{S_i})} \qquad (4.5)$$

where,

$$P_{r_{S_i}} = 1 - e^{-(k_{S_i} r_i)}, \quad k_{S_i} > 0 \qquad (4.3)$$

$$d{*}_i = x_i \sin\theta - y_i \cos\theta, \qquad (4.1)$$

Σ_{S_i} is the ASW range factor, and

θ is the approach of the ASW target.

4.2.4 PROBABILITY DISTRIBUTIONS OF THE TARGETS

Both the AAW and the ASW targets are uniformly distributed over known sectors. Let (a,b) represent such a sector. Let $f(\theta)$ be the probability density function of an AAW or an ASW target. Then $f(\theta)$ is given by

$$f(\theta) = 1/(b - a) \qquad a \leq \theta \leq b \qquad (4.6)$$

$$= 0 \qquad \text{otherwise}$$

4.2.5 SURVIVAL PROBABILITY OF THE CARRIER

The probability that the carrier survives is the probability that at least one ship kills the AAW target in the AAW sector (a,b) and at least one ship kills the ASW target in the ASW sector (c,d).

$$\text{Prob(carrier survives)} = \int_a^b f(\theta) \quad \left(1 - \prod_{i=1}^{2}(1-P_{A_i})\right) d\theta$$
$$\times \int_c^d f(\theta) \quad \left(1 - \prod_{i=1}^{2}(1-P_{S_i})\right) d\theta \tag{4.7}$$

4.3 AN AAW AND AN ASW SHIP - NUMERICAL RESULTS

We wish to test the sensitivity of the model and observe how the two ships work in conjunction with each other in order to maximize the probability of survival of the carrier, under various scenarios. We test for symmetry and consider the result of a pure (AAW or ASW but not both) threat. We examine the effects of multiple targets and different sectors on the objective function and on the optimal positions of the ships. We also study the limiting cases.

We use the following notation throughout the entire chapter.

NOTATION

The maximum probability of carrier survival = P_f .

The optimal position of the AAW ship from the carrier, in polar coordinates, is: (R_1, ϕ_1).

The optimal position of the ASW ship from the carrier, in polar coordinates, is: (R_2, ϕ_2).

4.3.1 SUITABLE K_{A_i} , K_{S_i} , Z_{A_i} , AND Z_{S_i} VALUES

In Experiments 3.11 and 3.12 of Chapter 3, we discovered appropriate parameter values to represent standard AAW and ASW ships respectively. They are shown in Tables 4.0a and 4.0b.

These canonical values will be used throughout this chapter, unless otherwise specified.

Note that the ASW ship has excellent AAW capability (almost as good as the AAW ship). However, the AAW ship has poor ASW capabilities in relation to those of the ASW ship.

$\underline{k}_{A1}$	$\underline{k}_{S1}$	$\underline{\Sigma}_{A1}$	$\underline{\Sigma}_{S1}$
0.2	0.1	1000	500

Table 4.0a Canonical AAW Ship Parameter Values

$\underline{k}_{A2}$	$\underline{k}_{S2}$	$\underline{\Sigma}_{A2}$	$\underline{\Sigma}_{S2}$
0.2	0.1	850	8000

Table 4.0b Canonical ASW Ship Parameter Values

4.3.2 SINGLE SHIP - SINGLE THREAT CASE

We return for a moment to the single ship - single threat case of Chapter 3, in order that we may use it as a point of comparison. The survival probability of the carrier in Experiments 4.1 and 4.2 is given by Eq'ns (3.5) and (3.7) for AAW and ASW threats respectively. In Experiment 4.1, we seek the values of P_f and (R_1, ϕ_1) when a single AAW ship is required to defend the carrier against an AAW threat in a sector extending from (0, 60). Experiment 4.2 is analogous, with the AAW ship replaced by an ASW ship.

EXPERIMENT 4.1

Purpose: Given the canonical k_{A_1} and Σ_{A_1} values for the AAW ship (Table 4.0a) and a (0,60) sector with a single AAW target, to determine P_f and (R_1, ϕ_1).

Table 4.10 shows the value of the fixed parameters in Experiment 4.1. Table 4.11 shows the values of P_f and (R_1, ϕ_1).

DISCUSSION OF RESULTS (Table 4.11)

Because of the uniform disribution of the target, ϕ_1 = 30; R_1 = 20.4 and P_f = .949 are both satisfactory.

Number of Ships	Type of Ship	Number of Sectors	Type of Sector	Sector	Number of Targets
1	AAW	1	AAW	0-60	1

Table 4.10 Fixed Parameters in Exp't 4.1: 1 Ship

Surv. Prob. P_f	.949
Ship Range R_1	20.40
Ship Angle ϕ_1	30

Table 4.11 Survival Probability and AAW Ship Location

EXPERIMENT 4.2

Purpose: Given the canonical values of k_{S_2} and Σ_{S_2} for an ASW ship (Table 4.0b) and a (0,60) sector with a single ASW threat, to determine the values of P_ϵ and (R_2,ϕ_2).

Table 4.20 shows the values of the fixed parameters in Experiment 4.2. Table 4.21 shows the values of P_ϵ and (R_2,ϕ_2).

DISCUSSION OF RESULTS (Table 4.21)

Because of the uniform distribution of the target, ϕ_2= 30; R_2= 46.31 and P_ϵ = .968 are both satisfactory.

4.3.3 SYMMETRY

It is worthwhile and interesting to discover what role symmetry plays in the model. The next experiment uses different initial conditions to determine how the optimal ship-to-carrier distances R_1 and R_2, and their associated angles ϕ_1 and ϕ_2 are affected.

Number of Ships	Type of Ship	Number of Sectors	Type of Sector	Sector	Number of Targets
1	ASW	1	ASW	0-60	1

Table 4.20 Fixed Parameters in Exp't 4.2: 1 Ship

Surv. Prob. P_f	.968
Ship Range R_2	46.41
Ship Angle ϕ_2	30

Table 4.21 Survival Probability and ASW Ship Location

EXPERIMENT 4.3

Purpose: To identify under what conditions we may expect symmetry. We use the parameter values in Tables 4.0a and 4.0b, and identical (0,60) AAW and ASW threat sectors, each with a single target.

Table 4.30 shows the values of the fixed parameters in Experiment 4.3. Table 4.31 shows P_f , (R_1,ϕ_1) and (R_2,ϕ_2) as a result of different initial conditions. (The initial conditions are in Cartesian coordinates.)

DISCUSSION OF RESULTS (Table 4.31)

As we would expect, there is symmetry about the 30° line. See Figures 4.1a and 4.1b.

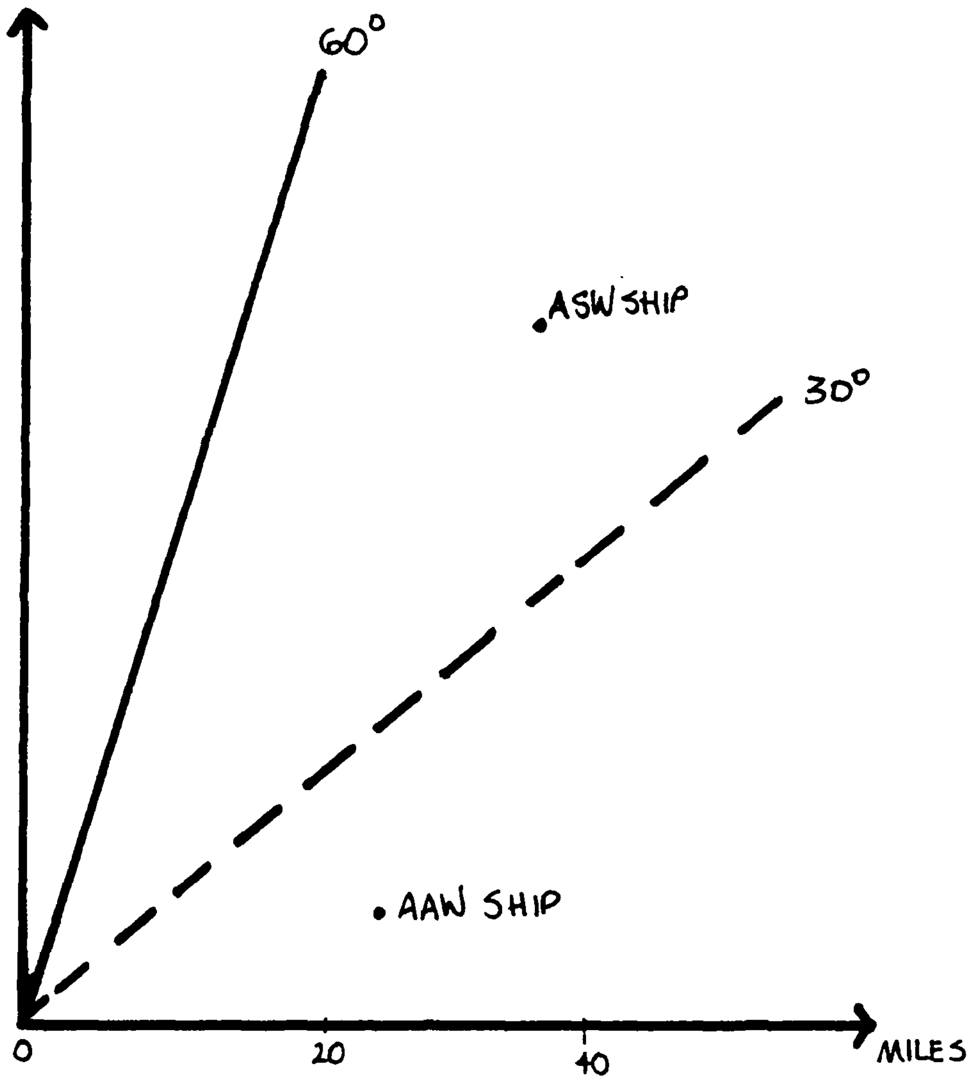
This is apparent when we compare the first two cases of Table 4.31. In the first case, the AAW ship is below the 30° line with initial coordinates (20,5), while the ASW ship is above the 30° line with initial coordinates (30,40). The optimal locations reflect a small change in positon, and ϕ_1 = 15.02 and ϕ_2= 45.3 indicates that the AAW ship is still below the 30° line and that the ASW ship is still above the 30° line. R_1 = 24.51 and R_2= 38.85 are both desirable AAW and ASW locations respectively.

Number of Ships	Type of Ships	Number of Sectors	Type of Sectors	Sectors	Number of Targets
2	1 AAW	2	1 AAW	0-60	1 AAW
	1 ASW		1 ASW	0-60	1 ASW

Table 4.30 Fixed Parameters in Exp't 4.3: 2 Ships

	Case 1	Case 2	Case 3
Init. Cond'n	(20,5) AAW	(50,50) AAW	(20,5) AAW
(Ship Position)	(30,40) ASW	(10,5) ASW	(40,10) ASW
Surv. Prob. P_f	.986	.986	.986
(R_1, ϕ_1)	(24.51,15.02)	(24.51,44.98)	(24.51,44.98)
(R_2, ϕ_2)	(38.85,45.30)	(38.85,14.70)	(38.85,14.70)

Table 4.31 Survival Probability and Ship Locations



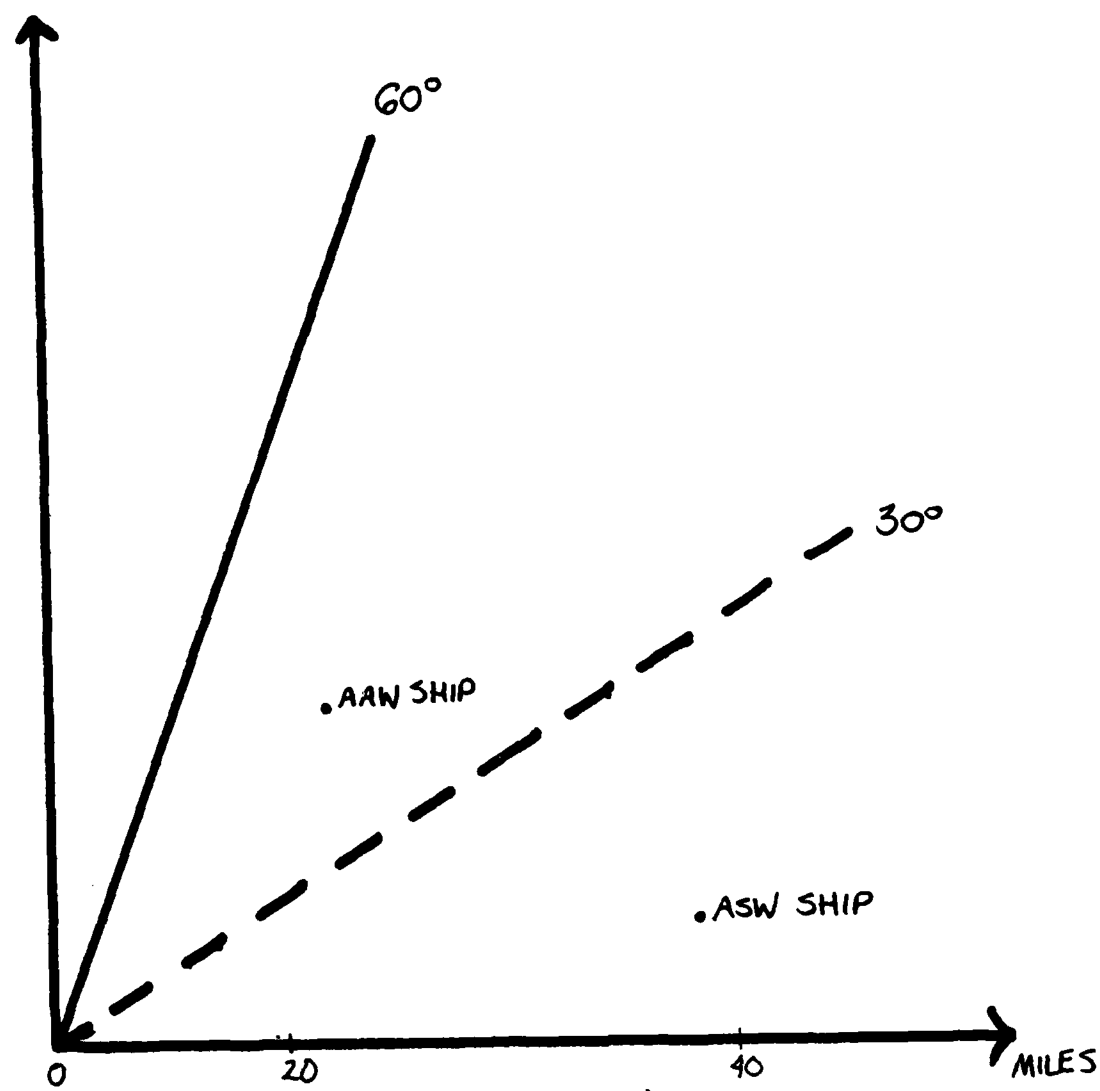
Legend

1) Two Identical Threat Sectors (0 ,60): One AAW, One ASW

2) Location of AAW Ship: (24.51, 15.02°)

3) Location of ASW Ship: (38.85, 45.3°)

Figure 4.1a Example of Symmetry (along with Figure 4.1b)



Legend

1) Two Identical Threat Sectors (0 ,60): One AAW, One ASW

2) Location of AAW Ship: (24.51, 44.98°)

3) Location of ASW Ship: (38.85, 14.7°)

Figure 4.1b Example of Symmetry (along with Figure 4.1a)

In the second case, we use an initial condition of (50,50) for the AAW ship, and (10,5) for the ASW ship. Now the AAW ship is on the 45^{o} line, and the ASW ship is below the 30^{o} line. The final result of $(24.51,44.98^{o})$ for the AAW ship, and $(38.85,14.7^{o})$ for the ASW ship demonstrates that the image was produced across the 30^{o} line, and therefore we have symmetry. It is also interesting to note that, given the initial conditions, the model brought in the AAW ship and pulled out the ASW ship as was desired. The final objective function value P_f is, of course, the same in both cases.

4.3.4 PURE THREATS - TWO SHIPS

We now turn to the situation in which only one type of threat exists - AAW or ASW, but not both. The carrier is still, however, being defended by both the AAW and the ASW ship. This scenario is of interest since it may shed light on the behaviour of the model under multiple AAW (or ASW, but not both) threats. For example, if the number of AAW threats is increased to 100, and there is a single ASW threat, do the optimal ship-to-carrier locations (R_1,ϕ_1) and (R_2,ϕ_2) converge to those of the situation in which there is only a pure AAW threat? In order to answer this question, we need the results of (R_1,ϕ_1) and (R_2,ϕ_2) for the pure threat; hence, the following two experiments. It is sufficient to run the experiments with only one target, since we saw in Claim 3.1

that the optimal ship-to-carrier distance is independent of the number of targets.

EXPERIMENT 4.4

Purpose: To obtain the values of (R_1, ϕ_1) and (R_2, ϕ_2) for a pure AAW threat. There is only one (0,60) sector with a single AAW threat.

Table 4.40 shows the values of the fixed parameters of Experiment 4.4. Table 4.41 contains the results of Experiment 4.4; P_f , (R_1, ϕ_1) and (R_2, ϕ_2).

DISCUSSION OF RESULTS (Table 4.41)

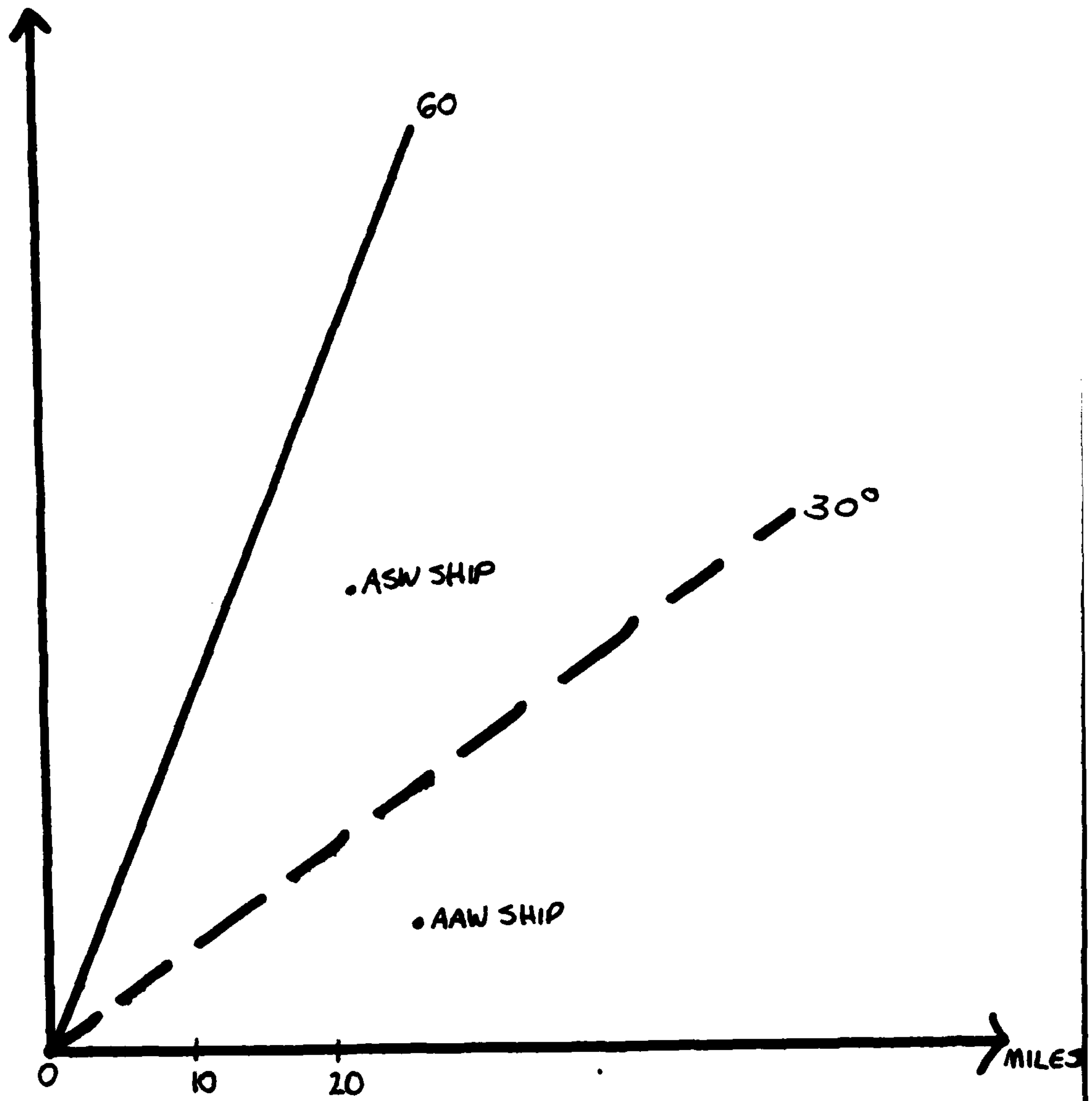
Because both the AAW and the ASW ship have excellent AAW capabilities ($\Sigma_{A_1} = 1000, \Sigma_{A_2} = 850$), the survival probability, P_f , is very high. $R_1 = 22.10$ and $R_2 = 21.49$ both reflect typical AAW ship locations, i.e. locations which are close to the carrier. $\phi_1 = 15.21^\circ$ and $\phi_2 = 44.91^\circ$ reveal that the ships are well positioned about the 30° line. We know that, given different initial conditions, we could obtain symmetric optimal locations,(results of Experiment 4.3). See Figure 4.2.

Number of Ships	Type of Ships	Number of Sectors	Type of Sector	Sector	Number of Targets
2	1 AAW	1	AAW	0-60	1
	1 ASW				

Table 4.40 Fixed Parameters in Exp't 4.4: 2 Ships

Surv. Prob. P_f	.997
(R_1, ϕ_1)	(22.10,15.21)
(R_2, ϕ_2)	(21.49,44.91)

Table 4.41 Survival Probability and Ship Locations: Pure AAW Threat



<u>Legend</u>

1) Location of AAW Ship: (22.10, 15.21°)

2) Location of ASW Ship: (21.49, 44.91°)

3) Threat Sector (0, 60)

Figure 4.2 Locations of Ships for Pure AAW Threat

EXPERIMENT 4.5

Purpose: To obtain the values of (R_1, ϕ_1) and (R_2, ϕ_2) for a pure ASW threat in a (0,60) sector.

Table 4.50 shows the values of the fixed parameters in Experiment 4.5. Table 4.51 contains the results of Experiment 4.5; P_{ϵ}, (R_1, ϕ_1), and (R_2, ϕ_2).

DISCUSSION OF RESULTS (Table 4.51)

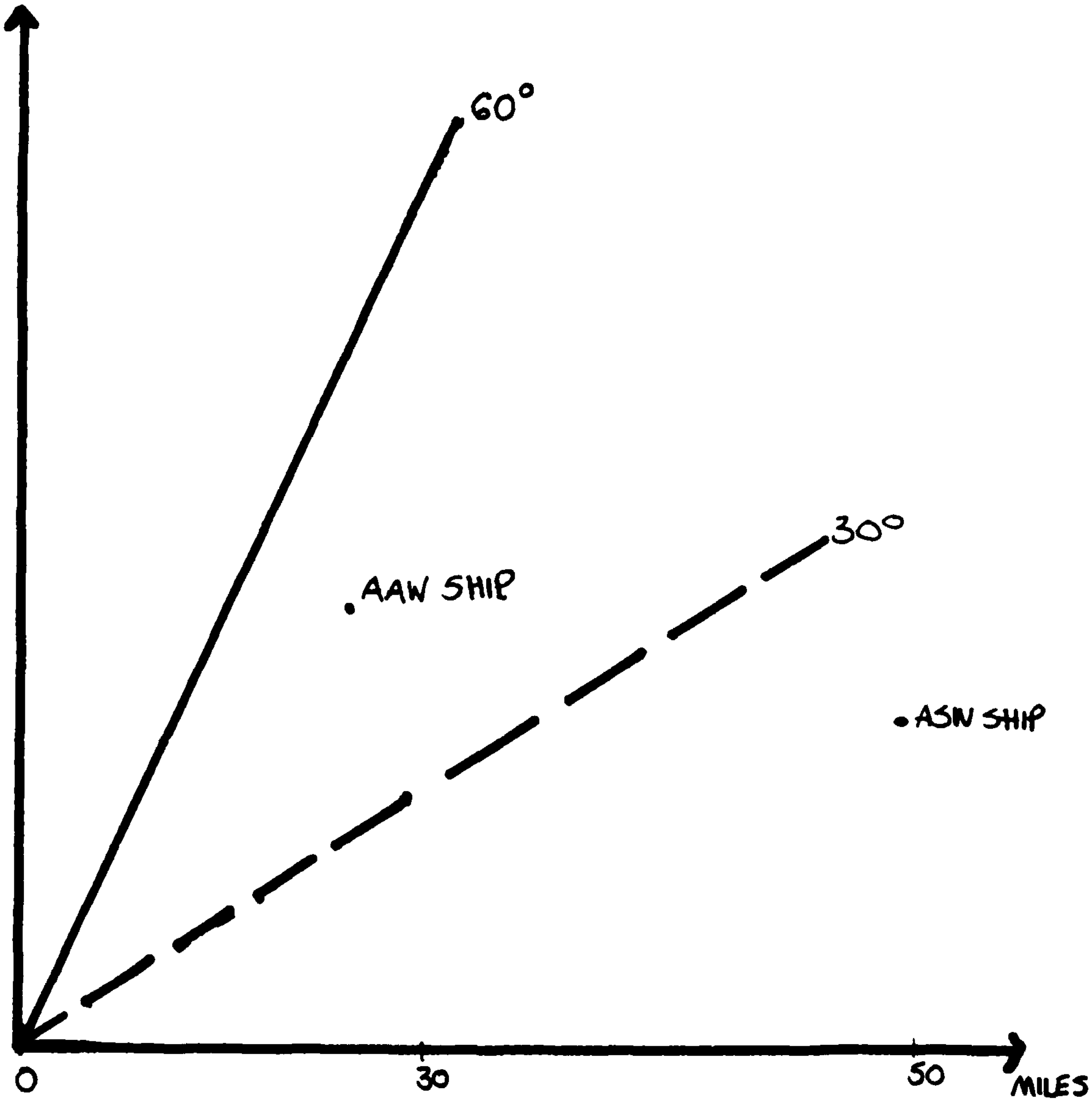
The ASW ship has an extremely high ASW capability and this is reflected in a high survival probability. Its optimal ship-to-carrier distance, R_2, is typical of that of an ASW ship in that it is relatively far from the carrier. In contrast, the AAW ship has a fairly poor ASW capability, (Σ_{S_1} = 500) and its optimal ship-to-carrier distance represents that of an AAW ship, suggesting that the large survival probability is largely due to the ASW ship. Both ships are, nevertheless, well positioned about the 30° line. See Figure 4.3.

Number of Ships	Type of Ships	Number of Sectors	Type of Sector	Sector	Number of Targets
2	1 AAW	1	1 ASW	0-60	1
	1 ASW				

Table 4.50 Fixed Paramaters in Exp't 4.5: 2 Ships

Surv. Prob. P_f	.934
(R_1, ϕ_1)	(29.40,45.98)
(R_2, ϕ_2)	(49.10,16.92)

Table 4.51 Survival Probability and Ship Locations: Pure ASW Threat



Legend

1) Location of AAW Ship: (29.4, 45.98°)

2) Location of ASW Ship: (49.1, 16.92°)

3) Threat Sector (0 , 60)

Figure 4.3 Locations of Ships for Pure ASW Threat

4.3.5 MULTIPLE TARGETS, LIMITING CASES, AND A CHANGE IN SECTOR

In this section, we examine the consequences of multiple targets, probe the limiting cases and study the results of a change in sector. We want to observe how the two ships interact to produce the maximum probability of survival of the carrier under these types of scenarios.

EXPERIMENT 4.6

Purpose: To obtain the optimal locations, i.e. the values of (R_1, ϕ_1) and (R_2, ϕ_2) for different numbers of AAW threats, while the number of ASW threats is held constant at one. The AAW threat sector and the ASW threat sector both extend from (0,60).

Table 4.60 shows the values of the fixed parameters of Experiment 4.6. Table 4.61 shows the effect on P_c, (R_1, ϕ_1) and (R_2, ϕ_2) of changing the AAW threat concentration.

DISCUSSION OF RESULTS (Table 4.61)

The results of the experiment show that as the number of AAW targets increase, R_1 and R_2 increase, as do ϕ_1 and ϕ_2. This information is particularly interesting when we compare it to the results of Experiment 4.4. In the limiting case, when the number of AAW targets is "large", the single ASW target is of little importance, as the two ships virtually confine their intentions to the

Number of Ships	Type of Ships	Number of Sectors	Type of Sectors	Sectors	Number of ASW Targets
2	1 AAW	2	1 AAW	0-60	1
	1 ASW		1 ASW	0-60	

Table 4.60 Fixed Parameters in Exp't 4.6: 2 Ships

Number of AAW Targets	P_f	(R_1,ϕ_1)	(R_2,ϕ_2)
1	.985	(24.51,44.98)	(38.80,14.70)
2	.980	(24.00,45.00)	(35.00,14.40)
3	.975	(23.80,45.10)	(32.50,14.38)
6	.962	(23.40,45.10)	(28.60,14.41)
12	.940	(23.00,45.00)	(25.50,14.50)
25	.899	(22.70,44.97)	(23.60,14.73)
50	.827	(22.48,44.90)	(22.60,14.87)

Table 4.61 Effect of Varying the Number of AAW Targets

massive AAW threat. The optimal survival probability, P_f , is almost completely determined by the AAW capability of each ship, Σ_{A_1} and Σ_{A_2}. We see by the AAW capability of each ship, Σ_{A_1} and Σ_{A_2} . We see that $(R_1,\phi_1)$and (R_2,ϕ_2) of Experiment 4.6 converge to (R_1,ϕ_1) and (R_2,ϕ_2) of Experiment 4.4, respectively, in the limit.

Note that in this experiment, the major effect of increasing the number of AAW targets is in the location of the ASW ship. For increasing number of targets, the ASW ship is being pulled more and more toward the carrier. Very minor changes occur in the location of the AAW ship. This is due to the excellent AAW capability of the ASW ship.

EXPERIMENT 4.7

Purpose: To obtain the values of (R_1,ϕ_1) and (R_2,ϕ_2) under different ASW target densities, when the number of AAW targets is held constant at one. The AAW and the ASW threat sectors both extend from (0,60).

Table 4.70 contains the values of the fixed parameters of Experiment 4.7. Table 4.71 shows the effect on P_f , (R_1,ϕ_1) and (R_2,ϕ_2) of changing the number of ASW targets.

Number of Ships	Type of Ships	Number of Sectors	Type of Sectors	Sectors	Number of AAW Targets
2	1 AAW	2	1 AAW	0-60	1
	1 ASW		1 ASW	0-60	

Table 4.70 Fixed Parameters in Exp't 4.7: 2 Ships

Number of ASW Targets	P_f	(R_1, ϕ_1)	(R_2, ϕ_2)
1	.985	(24.51,44.98)	(38.85,14.70)
2	.979	(25.26,44.97)	(42.11,15.20)
3	.972	(25.84,45.02)	(43.62,15.51)
6	.953	(26.85,45.18)	(45.65,16.04)
12	.917	(27.70,45.40)	(47.00,16.40)
25	.844	(28.40,45.60)	(47.90,16.60)
50	.721	(28.90,45.70)	(48.50,16.70)
100	.527	(29.10,45.80)	(48.70,16.80)

Table 4.71 Effect of Varying the Number of ASW Targets

DISCUSSION OF RESULTS (Table 4.71)

It is no surprise that we will draw the same conclusions here that we did for Experiment 4.6. Again, (R_1, ϕ_1) and (R_2, ϕ_2) of this experiment converge to (R_1, ϕ_1) and (R_2, ϕ_2) of Experiment 4.5, respectively, in the limit, as the number of ASW targets gets large. This is so because as the ASW threat concentration increases, the AAW threat becomes negligible, and the two ships are required to flex their ASW capability muscles.

In this experiment, as the number of ASW targets increases, both ships are moved away from the carrier in a coordinated manner. Even though the AAW ship has limited ASW capabilities, they are still being effectively utilized.

Next, we see what happens when.the ASW sector is shifted.

EXPERIMENT 4.8

Purpose: To see the effect on P_f , (R_1, ϕ_1) and (R_2, ϕ_2) of shifting the ASW sector. The AAW sector does not change and extends from (0,60). Both the AAW sector and the ASW sector have a single target threat.

Table 4.80 shows the values of the fixed parameters of Experiment 4.8. Table 4.81 shows the effect on P_f , (R_1, ϕ_1) and (R_2, ϕ_2) of changing the ASW sector.

Number of Ships	Type of Ships	Number of Sectors	Type of Sectors	AAW Sector	Number of Targets
2	1 AAW	2	1 AAW	0-60	1 AAW
	1 ASW		1 ASW		1 ASW

Table 4.80 Fixed Parameters in Exp't 4.8: 2 Ships

ASW Sector	P_f	(R_1, ϕ_1)	(R_2, ϕ_2)
0-60	.985	(24.51,15.02)	(38.80,45.30)
10-70	.984	(23.41,16.84)	(40.81,47.69)
20-80	.981	(21.91,18.26)	(40.61,50.68)
30-90	.975	(20.60,19.56)	(39.29,54.51)

Table 4.81 Effect of Changing the ASW Threat Sector

DISCUSSION OF RESULTS (Table 4.81)

As the ASW sector was shifted from (0,60) to (30,90), the AAW ship is being pulled in toward the carrier and moves more and more toward the center of the AAW sector, to provide better AAW coverage. The range of the ASW ship does not change much; its angle changes significantly moving it toward the center of the ASW threat sector. However, the ASW ship still contributes to the AAW defense. A decrease in the optimal survival probability, P_f, as the threat sector area expands, also supports the theory that the two ships cannot defend the carrier as well.

What happens when we increase the number of AAW targets under this type of scenario? This is the subject of the next experiment.

EXPERIMENT 4.9

Purpose: To determine the effect on (R_1,ϕ_1) and (R_2,ϕ_2) of increasing the number of AAW targets, while keeping the ASW target constant at one. The AAW sector extends from (0,60) and the ASW sector is from (30,90).

Table 4.90 contains the values of the fixed parameters of Experiment 4.9. Table 4.91 demonstrates the effect of increasing the number of AAW targets.

Number of Ships	Type of Ships	Number of Sectors	Type of Sectors	Sectors	Number of ASW Targets
2	1 AAW	2	1 AAW	0-60 AAW	1
	1 ASW		1 ASW	30-90 ASW	

Table 4.90 Fixed Parameters in Exp't 4.9: 2 Ships

Number of AAW Targets	P_f	(R_1,ϕ_1)	(R_2,ϕ_2)
1	.975	(20.60,19.56)	(39.29,54.51)
2	.967	(21.04,17.32)	(36.38,50.55)
4	.955	(21.38,16.05)	(33.12,48.01)
8	.937	(21.61,15.41)	(29.65,46.53)
16	.907	(21.75,15.18)	(26.56,45.71)
32	.856	(21.85,15.14)	(24.32,45.28)

Table 4.91 Effect of Varying the Number of AAW Targets

DISCUSSION OF RESULTS (Table 4.91)

As the AAW threat becomes large, the ASW threat becomes negligible. The optimal positions of the ships are dictated essentially by the existence of the AAW threat, and we notice that (R_1,ϕ_1) and (R_2,ϕ_2) of this experiment converge to (R_1,ϕ_1) and (R_2,ϕ_2) of Experiment 4.5, (pure AAW threat) respectively, in the limit as the number of AAW targets gets large.

Note that the range of the AAW ship does not change significantly; however, its location shifts to protect the lower half of the AAW threat sector. The ASW ship is pulled close to the carrier and moves to protect the upper half of the AAW threat sector.

4.3.6 THREE SHIPS

We take a glimpse at how the model operates when confronted with three ships protecting the carrier. We choose to observe its behaviour with two AAW ships and one ASW ship. In the succeeding experiments, we consider a single AAW sector extending from (0,60) and a single ASW sector extending from (30,90). We want to establish how the three ships interact in order to produce the maximum survival probability of the carrier, given different target densities. The ASW ship and one of the AAW ships have the parameter values of Tables 4.0a and

4.0b respectively. For the second AAW ship, we see the effect of varying two of its parameters.

The pertinent mathematical equations follow directly from Section 2.2.3.1 Case 4.

NOTATION

The optimal ship-to-carrier location of the second AAW ship = (R_3, ϕ_3)

EXPERIMENT 4.10

Purpose: To see the effect on P_f, (R_1, ϕ_1), (R_2, ϕ_2) and (R_3, ϕ_3) of increasing the number of ASW targets, when the second AAW ship has essentially no ASW capability.

Table 4.100 shows the values of the fixed parameters of Experiment 4.10. Table 4.101 shows the result of increasing the ASW target. concentration.

Number of Ships	Type of Ships	Number of Sectors	Type of Sectors	Sectors	Number of Targets	k_{A_S}	k_{S_S}	Σ_{A_S}	Σ_{S_S}
3	2 AAW	2	1 AAW	0-60 AAW	1	.2	0	1000	10
	1 ASW		1 ASW	30-90 ASW					

Table 4.100 Fixed Parameters in Exp't 4.10: 3 Ships

No. of ASW Targets	P_F	(R_1, ϕ_1)	(R_2, ϕ_2)	(R_3, ϕ_3)
1	.992	(25.29,43.33)	(46.39,69.82)	(25.13,11.42)
10	.937	(28.79,43.97)	(48.76,72.73)	(24.98,11.67)
20	.880	(29.09,43.99)	(48.93,72.91)	(24.98,11.68)

Table 4.101 Effect of Varying the Number of ASW Targets

DISCUSSION OF RESULTS (Table 4.101)

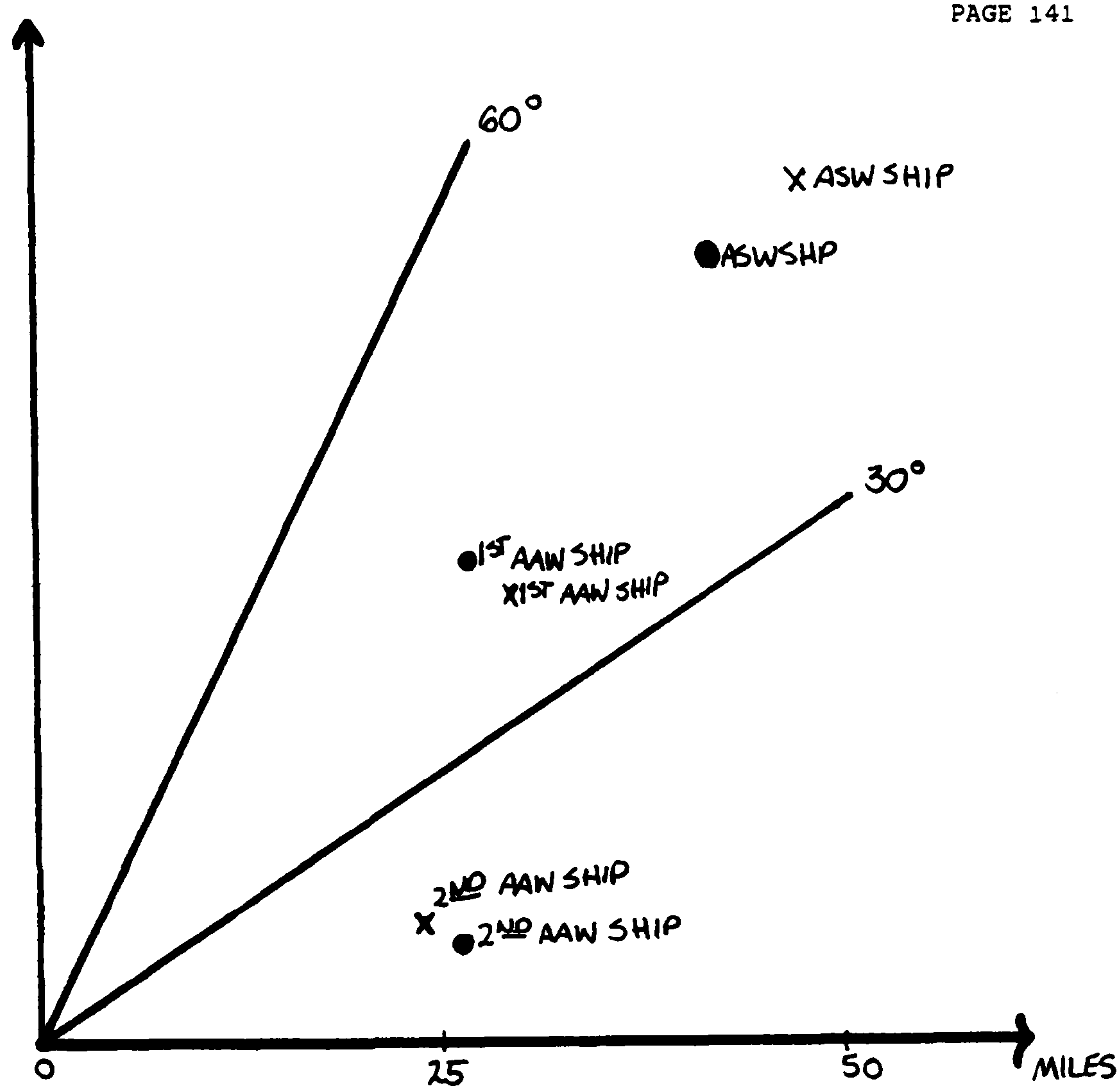
As the number of ASW targets increases from one to ten, the first AAW ship is called to the aid of the ASW ship; it is moved out and its angle is increased, as its ASW capability becomes more useful. The second AAW ship, due to its extremely poor ASW performance, is hardly moved at all. As the number of ASW targets is yet further augmented to twenty, we witness even less of a change in optimal positions, suggesting that we are converging to the best locations. See Figure 4.4.

Next, we improve the ASW capability of the second AAW ship .

EXPERIMENT 4.11

Purpose: To see the effect on P_f , (R_1,ϕ_1), (R_2,ϕ_2) and (R_3,ϕ_3) of increasing the number of ASW targets, when the second AAW ship has a good ASW capability, (better than that of the first AAW ship).

Table 4.110 shows the values of the fixed parameters of Experiment 4.11. Table 4.111 shows the effect on P_f , (R_1,ϕ_1), (R_2,ϕ_2) and (R_3,ϕ_3) of increasing the ASW target density.



<u>Legend</u>

1)● Location of First AAW Ship: (25.29, 43.33°) (1 Target)

2)● Location of ASW Ship: (46.39, 69.82°) (1 Target)

3)● Location of Second AAW Ship: (25.13, 11.42°) (1 Target)

4)X Location of First AAW Ship: (29.09, 43.99°) (20 Targets)

5)X Location of ASW Ship: (48.93, 72.91°) (20 Targets)

6)X Location of Second AAW Ship: (24.98, 11.68°) (20 Targets)

Figure 4.4 Varying the Number of ASW Targets: Three Ships

Number of Ships	Type of Ships	Number of Sectors	Type of Sectors	Sectors	Number of Targets	k_{A_3}	k_{S_3}	Σ_{A_3}	Σ_{S_3}
3	2 AAW	2	1 AAW	0-60 AAW	1 AAW	.2	.1	1000	3000
	1 ASW		1 ASW	30-90 ASW					

Table 4.110 Fixed Parameters in Exp't 4.11: 3 Ships

No. of ASW Targets	P_f	(R_1,ϕ_1)	(R_2,ϕ_2)	(R_3,ϕ_3)
1	.997	(21.08,13.10)	(43.66,63.95)	(25.29,35.47)
10	.987	(20.90,18.49)	(49.14,74.39)	(33.84,45.30)
20	.980	(20.59,22.50)	(49.93,75.69)	(35.66,46.87)

Table 4.111 Effect of Varying the Number of ASW Targets

DISCUSSION OF RESULTS (Table 4.11)

Immediately we experience the effect of giving the second AAW ship a powerful ASW capability. Even when there is only one AAW and one ASW threat, the second AAW ship is thrown into the ASW threat sector ($\phi_3 = 35.47$) emphasizing its superior ASW capability. The first AAW ship with its significantly weaker ASW capability, is brought deeper into the AAW sector ($\phi_1 = 13.10$).

It is useful to compare the results of Experiments 4.10 and 4.11 in the cases where there is only one AAW and one ASW target. In Experiment 4.10, the first AAW ship acts to help the ASW ship because the second AAW ship has virtually no ASW capability. In Experiment 4.11, the roles of the two AAW ships are reversed, because the second AAW ship has a better ASW capability. Also, when the number of ASW targets is increased from one to twenty, R_3 increases substantially, demonstrating that the second AAW ship is effectively serving as an ASW ship. Its angle, ϕ_3, is always increasing, bringing it more towards the ASW sector.

It is worthwhile to note that, in Experiment 4.11, the optimal survival probability, P_f, barely suffers as the ASW targets become thicker. It decreases from .997 (one target) to .980 (twenty targets). On the other hand, the lack of ASW capability in Experiment 4.10, is more adequately represented, as P_f plummets from .992 (one target) to .880 (twenty targets).

EXPERIMENT 4.12

Purpose: To see the effect on P_f , (R_1, ϕ_1), (R_2, ϕ_2) and (R_3, ϕ_3) of augmenting the number of AAW targets.

Table 4.120 shows the values of the fixed parameters of Experiment 4.12. Table 4.121 shows the result of increasing the number of AAW targets.

DISCUSSION OF RESULTS (Table 4.121)

The ASW ship has an excellent AAW capability. As soon as the AAW threat becomes more prominent, the ASW ship is brought down towards the AAW sector (ϕ_2 is decreasing). It is placed closer to the carrier, taking on the optimal ship-to-carrier distance, R_2 , of an AAW ship. Because all three ships are equipped with good AAW capabilities, the optimal survival probability remains excellent, (P_f = .990) even when the number of AAW targets is increased to twenty.

How good are the results when there are three threat sectors? As a final experiment, we consider this possibility.

Number of Ships	Type of Ships	Number of Sectors	Type of Sectors	Sectors	Number of Targets	k_{A_3}	k_{S_3}	Σ_{A_3}	Σ_{S_3}
3	2 AAW	2	1 AAW	0-60 AAW	1 ASW	.2	.1	1000	3000
	1 ASW		1 ASW	30-90 ASW					

Table 4.120 Fixed Parameters in Exp't 4.12: 3 Ships

No. of AAW Targets	P_f	(R_1,ϕ_1)	(R_2,ϕ_2)	(R_3,ϕ_3)
1	.997	(21.08,13.10)	(43.66,63.95)	(25.29.35.47)
10	.993	(21.61,10.84)	(35.59,53.05)	(23.20,26.15)
20	.990	(21.40,11.43)	(32.09,51.98)	(22.61,24.33)

Table 4.121 Effect of Varying the Number of AAW Targets

EXPERIMENT 4.13

Purpose: To see the effect on P_f , (R_1,ϕ_1), (R_2,ϕ_2) and (R_3,ϕ_3) when the number of ASW targets is augmented. The ASW sector extends from (60,90), and the two AAW sectors extend from (0,20) and (30,50).

Table 4.130 shows the values of the fixed parameters of Experiment 4.13. Table 4.131 shows the results of Experiment 4.13.

DISCUSSION OF RESULTS (Table 4.131)

For a single AAW and ASW threat, the model successfully places an AAW ship in each AAW sector, and the ASW ship in the ASW sector. It also wisely places each ship roughly in the middle of its sector. The survival probability is high; P_f = .996. When the ASW target threat is increased to twenty, all the ships are shifted upwards toward the ASW sector. The optimal survival probability decreases (P_f = .978).

4.4 CONCLUDING REMARKS

In this chapter, we performed numerical experiments to investigate how two and three ships work in conjunction with one another in order to maximize the survival probability of the carrier.

Number of Ships	Type of Ships	Number of Sectors	Type of Sectors	Sectors	Number of Targets	$\underline{k}_{A_3}$	$\underline{k}_{S_3}$	$\underline{\Sigma}_{A_3}$	$\underline{\Sigma}_{S_3}$
3	2 AAW	3	2 AAW	0-20 AAW	1 AAW	.2	.1	1000	3000
	1 ASW		1 ASW	30-50 AAW					
				60-90 ASW					

Table 4.130 Fixed parameters in Exp't 4.13: 3 Ships

No. of ASW Targets	P_f	(R_1, ϕ_1)	(R_2, ϕ_2)	(R_3, ϕ_3)
1	.996	(24.65, 9.51)	(48.30,74.06)	(24.70,38.55)
20	.978	(20.66,12.68)	(57.27,77.78)	(28.12,47.23)

Table 4.131 Effect of Varying the Number of ASW Targets

Chapter 5 Conclusions and Suggestions for Future Research

CHAPTER 5 CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

5.1 CONCLUSIONS

A simple model that can be used to carry out quantitative investigations of the optimal location of multi-warfare capable platforms to carry out the AAW and ASW functions in defense of a carrier, has been developed.

Even with the obvious limitations of our model, the results of the research suggest that the methodology developed can be used in conjunction with more realistic models of AAW and ASW capabilities; in this case it is possible to develop a valuable decision-aid.

In spite of its simplicity, the model captures correctly what should happen in scenarios in which the coordination of multi-warfare capable platforms is essential for superior defense against simultaneous AAW and ASW threats. This is primarily reflected in the fact that the optimal ship locations with respect to the carrier change in a reasonable and sensible manner as a function of:

a) the location and number of AAW and ASW threat sectors

b) the density of the AAW and the ASW targets

c) the individual AAW and ASW capabilities of each of the ships

The nature of this information is valuable for both planning and tactical situations involving modern Battle Groups.

The model is a complete self-contained unit; it is mathematically rigorous and gives consistent results. It is robust in the sense that "small" changes in the parameters lead to "sensible" changes in the objective function and the solution. It is very general and does not confine us to a narrow stream of information, but rather opens the door to investigate a broad cross-section of scenarios. We are free to select any combination and strength of AAW and ASW ships, the type and intensity of the targets, and the type, location and number of threat sectors.

A special advantage of the model is that it is user - friendly. It is extremely easy to start new experiments, and the results are clear. Of course, because we depend on numerical approximations via the optimization routine (SUMSNO) (5) and the integration routine (D01BAF) (6), the numerical complexity of our model is predetermined.

5.2 SUGGESTIONS FOR FUTURE RESEARCH

The intent of this section is to present natural modifications and exciting extensions of the model. We propose generalizing the sectors to include not only the permissible sectors, but also combinations of non-permissible and overlapping multiple sectors. We consider the possibility of diverse probability distributions of the targets. We recommend introducing the carrier as a ship with its own defense capabilities. Finally, we shift gears and regard the model in a whole other dimension. We suggest adapting it to the setting of a game, and create the conditions under which it can be used.

5.2.1 PERMISSIBLE AND NON-PERMISSIBLE AND MULTIPLE SECTORS

In Chapter 2, we developed a model to represent the survival probability of a carrier, given air and/or sub threats, and simple models of the probabilities. The problem was formulated in two dimensions, with the carrier at the origin. Our objective was to find an optimal location for all of the ships defending the carrier, with respect to the carrier. In Section 2.2.3.1, we considered two different types of threat sectors, and defined the notion of a PERMISSIBLE sector (DEF'N 2.9). Permissible sectors are ones in which any ship will attempt to kill a target BEFORE it passes through the origin.

A natural way to extend the ideas of this paper would be to incorporate the possibility of non-permissible sectors. As we saw in Section 2.2.3.1, a nec-

essary and sufficient condition for a permissible sector (a,b) is that, for a given ship i with coordinates (x_i, y_i) we have

$$d^*_i = x_i \sin\theta - y_i \cos\theta \qquad \text{for all } \theta \text{ , } a \leq \theta \leq b$$

We now wish to relax this condition, and use the more general definition of d^*_i, found in DEFINITION 2.8.

If we consider the most general scenario, like that of Case 4 at the end of Chapter 2, we need to know how the survival probability equation is affected by the existence of non-permissible sectors. First, at each iteration we must determine the value of d^*_i. In DEFINITION 2.8, we stated that

$$\begin{aligned} d^*_i &= x_i \sin\theta - y_i \cos\theta & \qquad \phi_1 \leq \phi_i \leq \phi_2 \\ &= r_i & \qquad \text{otherwise} \end{aligned}$$

where,

ϕ_i is defined in DEFINITION 2.6, and

ϕ_1 and ϕ_2 are defined in DEFINITION 2.7.

It is easy to incorporate this information into the computer program. It suffices to declare and define ϕ_i, ϕ_1, and ϕ_2 and to add, wherever necessary, a control statement of the form:

IF($\phi_1 \leq \phi_i \leq \phi_2$)

THEN $d^*_i = x_i \sin\theta - y_i \cos\theta$

ELSE $d^*_i = r_i$

The use of non-permissible sectors, and hence the inclusion of the generalized d^*_i, leads us to yet a further extension of the model. We are now free to admit multiple overlapping sectors. The ultimate generalization of the model in this way is:

Case 5: M Ships, N_i Independent and Identically Distributed AAW Targets in AAW Sector i, i = 1,G, Q_j Independent and Identically Distributed ASW Targets in ASW Sector j, j = 1,H.

Then the survival probability becomes:

$$\text{Prob(carrier survives)} = \prod_{i=1}^{G} \left\{ \int_{a_i}^{b_i} f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i}) \quad d\theta \right\}^{N_i}$$

$$\times \prod_{j=1}^{H} \left\{ \int_{c_j}^{d_j} f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{S_i}) \quad d\theta \right\}^{Q_j} \tag{5.1}$$

where,

P_{A_i} is as in Eq'n (2.5),

P_{S_i} is as in Eq'n (2.6),

(a_i, b_i) are the limits of the ith AAW sector, and

(c_j, d_j) are the limits of the jth ASW sector.

5.2.2 DIFFERENT PROBABILITY DISTRIBUTIONS OF TARGETS

The introduction of different probability distributions of the targets on a per sector basis, would add some flavour to the model. It might be meaningful to allow the target threat to be denser, say, in the middle of the sector, and wane at the boundaries. Then, instead of having a single $f(\theta)$ (a uniform distribution) for every sector, we would have a series of distributions, $f_1(\theta)$, $f_2(\theta)$, ..., $f_{G+H}(\theta)$, where G+H = total number of (AAW and ASW) sectors.

5.2.3 THE CARRIER AS A CONTRIBUTING SHIP

Our model precludes the assistance of the carrier itself. In a realistic situation, the carrier would be counted on to provide some defense of its own. We could furnish the carrier with its own AAW (Σ_A) and ASW (Σ_S) capabilities. The carrier itself would then be treated as another ship, with the additional stipulation that its location not be changed; i.e. the carrier remains at (0,0) and we maximize its overall survival probability only over the positions of the AAW and the ASW ships, as before.

5.2.4 INTRODUCTION OF GAME THEORY

Another way to expand this problem is to formulate the problem in terms of a game. We consider two commanders, one responsible for AAW ships, the other for ASW ships, in the spirit of the Naval CWC command - and - control doctrine.

This game-theoretic approach is appealing. Under this scheme, each commander has a local objective of maximizing the probability of survival of the carrier given an AAW or an ASW threat, depending on his warfare responsibility. They share the (common) global objective of maximizing the total survival probability of the carrier, given both types of threats.

The commander's decisions of where to place his ships in order to realize his goal, are a direct result of maximizing his objective function with respect to the positions of the ships which are in his control, while holding the positions of the other commander's ships fixed.

The game persists as, at each iteration, each commander attempts to maximize his objective function, given the previous decision of the other. The game terminates when we converge to an equilibrium. This is known as the Nash strategy.

Now, we state the conditions which guarantee the existence of a Nash solution.

Necessary and Sufficient Conditions for a Nash Solution:

If $u_i \in U_i$, U_i compact, convex, non-empty, $J_i(u_1, u_2, \ldots\ldots, u_n)$ concave in U_i, continuous, then there exists a Nash equilibrium.

We formulate these ideas mathematically. Let commander 1 be the commander responsible for the AAW threats, and hence the AAW ships. Let J_1 be his objective function. Then,

J_1 = Prob(carrier survives given an AAW threat)

$$= \prod_{i=1}^{G} \left\{ \int_{a_i}^{b_i} f(\theta) \quad (1 - \prod_{i=1}^{T} (1 - P_{A_i})) \, d\theta \right\}^{N_i} \qquad (5.2)$$

where,

P_{A_i} is as in Eq'n (2.5),

(a_i, b_i) are the limits of the ith AAW sector,

N_i is the number of AAW targets in the ith AAW sector, and

G is the number of AAW sectors.

Let this commander have M ships, with coordinates: (x_1, y_1), (x_2, y_2),.... ..,(x_M, y_M).

Similarly, let commander 2 be the commander responsible for the ASW threats, and therefore the ASW ships. Let J_2 be his objective function.

Then,

J_2 = Prob(carrier survives given an ASW threat)

$$= \prod_{i=1}^{H} \left\{ \int_{c_i}^{d_i} f(\theta) \quad (1 - \prod_{i=1}^{T}(1-P_{S_i} \quad)) \quad d\theta \right\}^{Q_i} \tag{5.3}$$

where,

P_{S_i} is as in Eq'n (2.6),

(c_i, d_i) are the limits of the ith ASW sector,

Q_i is the number of ASW targets in the ith ASW sector, and

H is the number of ASW sectors.

If there are a total of T ships, then since commander 1 has M ships, commander 2 has (T - M) ships, with coordinates: (x_{M+1}, y_{M+1}), (x_{M+2}, y_{M+2}),..., ,..(x_T, y_T).

Every ship has both AAW and ASW capabilities, and hence characteristic k_{A_i}, k_{S_i}, Σ_{A_i}, and Σ_{S_i} values. Therefore, any ship, whether under the responsibility and control of commander 1 or commander 2, contributes to both J_1 and J_2, just as it did in the centralized case. On the other hand, commander 1 only has control over the locations of his (AAW) ships. Consequently, he maximizes J_1 with respect to the positions of his ships, while keeping the positions of the ASW ships fixed. Similarly for the ASW commander.

The existing computer program for the centralized problem can be used as a prototype for the game. The latter requires the interplay of two such programs, with slight modifications. The program could be divided into two similar modules, one for each commander. The module for commander 1 would consist of the following:

1. the objective function, J_1
2. an array containing the location coordinates of his ships
3. an array containing the location coordinates of the ASW ships
4. a facility to separate the P_{A_i} , i = 1,M (the AAW ships) from the P_{S_i} , i = M+1,T (the ASW ships). When maximizing J_1 , it is precisely the M AAW ships and their effect on the P_{A_i} , i = 1,M that are being controlled. The (T - M) ASW ships contribute to J_1, but their positions cannot be changed by the AAW commander These positions are obtained from the (previous) decision of the ASW commander and are held fixed in the P_{A_i} , i = M+1,T. We then write J_1 as:

$$J_1 = \prod_{i=1}^{G} \left\{ \int_{a_i}^{b_i} f(\theta) \left(1 - \prod_{i=1}^{M}(1-P_{A_i}) \prod_{i=M+1}^{T}(1-P_{A_i})\right) d\theta \right\}^{N_i} \qquad (5.4)$$

where,

P_{A_i} is as in Eq'n (2.5),

N_i is the number of AAW targets in the ith AAW sector,

(a_i, b_i) are the limits of the ith AAW sector, and

G is the number of AAW sectors.

5. an optimization and integration routine as in the original program

6. a facility to pass freely the solution (i.e. the new coordinates of the AAW ships) of the optimization to commander 2, so that he may proceed with the maximization of J_2.

A parallel module is clearly required for commander 2.

It is convenient to present the game in terms of an algorithm:

Step 1. Each commander provides initial conditions for the positions of his ships.

Step 2. Commander 1 maximizes J_1 with respect to (x_1, y_1), ... (x_M, y_M), holding (x_{M+1}, y_{M+1}), ... (x_T, y_T) fixed, from commander 2's previous decision.

Step 3. Commander 2 maximizes J_2 with respect to (x_{M+1}, y_{M+1}), ... (x_T, y_T) holding (x_1, y_1), ... (x_M, y_M) fixed, from commander 1's previous decision.

Step 4. IF $\left\{(x_1, y_1), \ldots, (x_M, y_M)\right\}^{new} = \left\{(x_1, y_1), \ldots, (x_M, y_M)\right\}^{old}$

OR

IF $\left\{(x_{M+1}, y_{M+1}), \ldots, (x_T, y_T)\right\}^{new} = \left\{(x_{M+1}, y_{M+1}), \ldots, (x_T, y_T)\right\}^{old}$,

THEN STOP; we have converged to a Nash strategy and an equilibrium has been reached.

The optimal values of J_1 and J_2 are their current values. The optimum locations of the ships are the current values of (x_1, y_1), ..., (x_M, y_M), (x_{M+1}, y_{M+1}), ..., (x_T, y_T).

ELSE return to Step 2.

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APPENDIX A PROOF OF CLAIM 3.1

Proof of CLAIM 3.1:

We want to show that maximizing the survival probability of the carrier with respect to the locations of the ships, given M ships and 1 target, is the same as maximizing it for M ships and N targets. We consider both of these cases and show that they are the same.

Case 1: M Ships, 1 Target

We wish to maximixe:

$$\max_{x_i, y_i} \left\{ \int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1 - P_{A_i})) \quad d\theta \right\} \qquad i = 1,..,M$$

In other words, we want to solve for (x_i, y_i), $i = 1,\ldots,M$

$$\frac{\partial}{\partial x_1} \left\{ \int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1 - P_{A_i})) \quad d\theta \right\} = 0$$

$$\frac{\partial}{\partial y_1} \left\{ \int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1 - P_{A_i})) \quad d\theta \right\} = 0$$

$$\vdots$$

$$\frac{\partial}{\partial x_i} \left\{ \int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1 - P_{A_i})) \quad d\theta \right\} = 0$$

$$\frac{\partial}{\partial y_i} \left\{ \int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1 - P_{A_i})) \quad d\theta \right\} = 0$$

$$\vdots$$

$$\frac{\partial}{\partial x_M}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\} = 0$$

$$\frac{\partial}{\partial y_M}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\} = 0$$

Case 2: M Ships, N Targets

Here we want to maximize:

$$\max_{x_i, y_i} \left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\}^N \qquad i = 1,\ldots,M$$

In other words, we want to solve for (x_i, y_i), $i = 1,\ldots,M$

$$\frac{\partial}{\partial x_1}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\}^N = 0$$

$$\frac{\partial}{\partial y_1}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\}^N = 0$$

$$\vdots$$

$$\frac{\partial}{\partial x_i}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\}^N = 0$$

$$\frac{\partial}{\partial y_i}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\}^N = 0$$

$$\vdots$$

$$\frac{\partial}{\partial x_M}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\}^N = 0$$

$$\frac{\partial}{\partial y_M}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\}^N = 0$$

Let us consider, for example:

$$\frac{\partial}{\partial x_i}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i} \quad)) \quad d\theta\right\}^N = 0. \qquad \text{(A.1)}$$

We want to show that Eq'n (A.1) is the same as :

$$\frac{\partial}{\partial x_i}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i})) \quad d\theta\right\} \qquad = 0 \qquad \text{(A.2)}$$

So,

$$\frac{\partial}{\partial x_i}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i})) \quad d\theta\right\}^N \qquad = 0 \qquad \text{(A.1)}$$

$\Rightarrow$

$$N\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i})) \quad d\theta\right\}^{N-1} \times \frac{\partial}{\partial x_i}\left\{\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i})) \quad d\theta\right\} = 0 \qquad \text{(A.3)}$$

In order to show that Eq'n (A.3) produces the same solution as Eq'n (A.2) we consider 3 possibilities.

Either i) $N = 0$

or

ii) $\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i})) \quad d\theta \quad = 0$

or

iii) $\frac{\partial}{\partial x_i}\int_a^b f(\theta) \quad (1 - \prod_{i=1}^{M}(1-P_{A_i})) \quad d\theta \quad = 0$

If iii) holds, we are done. So, we need to show that i) and ii) cannot hold.

i) N is the number of targets, so $N \neq 0$.

ii) If $\int_a^b f(\theta) \ (1 - \prod_{i=1}^{M}(1-P_{A_i} \ )) \ d\theta \ = 0$, then

either a) $f(\theta) = 0$

or b) (a ,b) has zero area

or c) $(1 - \prod_{i=1}^{M}(1-P_{A_i} \ )) = 0$.

Then:

a) $f(\theta)$ is the uniform distribution, and so $f(\theta) \neq 0$, in the area of interest, i.e. $f(\theta) = 1/(b - a)$, $a \leq \theta \leq b$,

b) sector (a ,b) has positive, non-zero area,

and,

c) $(1 - \prod_{i=1}^{M}(1-P_{A_i} \ )) \neq 0$ since $P_{A_i} \neq 1 \quad \forall i$.

Therefore, iii) holds always, and Case 1 and Case 2 produce the same solution, and the optimal ship-to-carrier distance, R, is independent of the number of (independent and identically distributed) targets.

APPENDIX B ABOUT THE PROGRAM

B.1 HOW THE MODEL IS IMPLEMENTED

This section deals with the numerical implementation of the model. It explains how the computer program works.

All experiments were run at the Massachusetts Institute of Technology on an I.B.M. 370/168.

The cost of a typical optimization run in CPU time is 6 seconds. The associated dollar cost is $ 1.25.

B.1.1 THE MAIN PROGRAM, LIBRARY ROUTINES AND SUBROUTINES

The program makes use of several modules and subroutines. The main program has three tasks. It

1) calls an optimization routine (SUMSNO) to maximize the objective function,
2) calls a subroutine (READ3) to read the input data, and
3) prints the final positions of the ships.

SUMSNO (5) is a NONLIN library subroutine which minimizes general unconstrained objective functions of low enough order. We use it to find the optimal locations of the ships - i.e. those coordinates of the ships that maximize the probability of survival of the carrier. At each iteration, SUMSNO "tries out" a set of ships' coordinates and evaluates the objective function, i.e., the survival probability of the carrier. It uses an approximation to both the gradient and the Hessian.

We recall from Chapter 2, Section 2.3.2.1 Case 4, that the most general expression for the survival probability of the carrier in a sector (a,b) is:

$$\text{Prob(survival)} = \left\{ \int_a^b f(\theta)\ \left(1 - \prod_{i=1}^{M}(1-P_{A_i})\right) d\theta \right\}^N \times \left\{ \int_a^b f(\theta)\ \left(1 - \prod_{i=1}^{M}(1-P_{S_i})\right) d\theta \right\}^Q . \tag{B.1}$$

where,

M = number of ships,

N = number of AAW targets, and

Q = number of ASW targets

$f(\theta)$ is the uniform distribution of a target over a sector (a,b)

P_{A_i} is the Prob(ship i kills an AAW target)

P_{S_i} is the Prob(ship i kills an ASW target)

Since the probability in Eq'n (B.1) is the product of integrals, we need a subroutine that will evaluate the integrals numerically. D01BAF, a subroutine which is part of the NAG library, is used to do this. We invoke D01BAF, the integration subroutine, at each iteration, from QDRTF, the subroutine that evaluates the objective function. There are two function subprograms, AAWFN1 and ASWFN1 (for AAW and ASW sectors respectively) which are called by D01BAF each time it is necessary to integrate over a particular sector.

Please refer to the References (5) and (6) for more information on SUMSNO and D01BAF respectively.

B.1.2 THE DATA FILE

The data is stored in a data file called KSH DT. One modifies the data by editing the contents of KSH DT. The subroutine that reads the input, READ3, assumes a fixed line-by-line format for the input file. This implies that maintaining the format is crucial in order for the program to work correctly. A sample format of the data file is included.

This report lends itself to experiments in which the number of AAW (ASW) ships and/or sectors and/or targets is varied. The data sits in arrays whose dimensions may be easily altered. This requires adding (for augmenting the size of an array) or deleting (for reducing the size of an array) lines from KSH DT. It is in this situation that one must pay attention to the format. Please refer to the comments in the subroutine READ3 of the program for any further information. The computer program can be found in Appendix C.

B.1.3 THE ITERATION SUMMARY

SUMSNO supplies an iteration summary via an internal subroutine ITSUMU. ITSUMU provides basic information about the objective function, such as its current value at a particular iteration. It provides the optimal solution in Cartesian coordinates. Our computer program also gives the optimal solution in polar coordinates.

Since the commands to run the program are coded in an EXEC file, it is easy to use the program. The EXEC command is:

```
DFKG3
```

For a more detailed description of the program and an explanation of its parameters, we advise the reader to consult the program itself. It is documented extensively and may be found in Appendix C.

Appendix C The Computer Program and its Documentation

APPENDIX C THE COMPUTER PROGRAM AND ITS DOCUMENTATION

C.1 THE COMPUTER PROGRAM AND ITS DOCUMENTATION

The actual program follows.

```
C
C       * * *  PURPOSE OF THIS PROGRAM  * * *
C
C
C THE PURPOSE OF THIS PROGRAM IS TO MAXIMIZE THE PROBABILITY OF SURVIVAL
C
C OF A CARRIER, GIVEN AIR AND SUB THREATS. THE CARRIER HAS NO DEFENSIVE
C
C CAPABILITIES OF ITS OWN. IT IS DEFENDED BY AAW (ANTI-AIR WARFARE) AND
C
C ASW (ANTI-SUB WARFARE) SHIPS. THE USER SUPPLIES INPUT CONCERNING
C
C THE NUMBER OF SHIPS, THE CHARACTERISTIC PARAMETERS OF THE SHIPS, THE
C
C NUMBER, TYPE (AAW OR ASW) AND LOCATION OF THE THREAT SECTORS, AND THE
C
C NUMBER OF TARGETS. ALL THIS IS EXPLAINED IN GREATER DETAIL AS IT IS
C
C ENCOUNTERED IN THE PROGRAM.
C
C
C       * * *  TO RUN THE PROGRAM  * * *
C
C IN ORDER TO RUN THIS PROGRAM, ONE USES THE EXEC FILE CALLED:
C
C     1) DFKG3, FOR PRINTINGS OF INITIAL INPUT AND FINAL OUTPUT ONLY,
C
C
C IT IS INVOKED SIMPLY BY TYPING: DFKG3
C
C
C
C       * * *  GENERAL OVERVIEW  * * *
C
C
C THE PROGRAM IS DIVIDED INTO FIVE MODULES. THEY ARE:
C
C 1. MAIN PROGRAM:
C
C THE MAIN PROGRAM HAS FIVE FUNCTIONS. IT
C
C   I. CALLS READ3, THE SUBROUTINE THAT READS IN A THE INPUT PARAMETERS.
C
C  II. CONVERTS DEGREES TO RADIANS
C
```

```
C III. PRINTS INFORMATION CONCERNING THE INPUT PARAMETERS
C
C  IV. CALLS SUMSNO, THE OPTIMIZATION ROUTINE
C
C   V. COMPUTES (VIA SUMSNO) THE OPTIMAL SHIP-TO-CARRIER DISTANCES, R(I)
C
C      OF EACH SHIP I, AND THEIR ASSOCIATED OPTIMAL ANGLES. IT THEN
C
C      PRINTS THEM AT THE END OF THE OUTPUT.
C
C
C 2. SUBROUTINE QDRTF
C
C   I. EVALUATES THE OBJECTIVE FUNCTION, F; F = SURVIVAL PROBABILITY OF
C
C      THE CARRIER. SINCE SUMSNO ACTUALLY MINIMIZES F, AND WE WANT TO
C
C      MAXIMIZE OUR OBJECTIVE FUNCTION, WE MINIMIZE   (- F).
C
C  II. SINCE F IS THE PRODUCT OF INTEGRALS, WE NEED AN INTEGRATION
C
C      ROUTINE TO EVALUATE THE INTEGRAL(S). THIS IS DONE BY D01BAF.
C
C      D01BAF IS CALLED BY QDRTF.
C
C III. CALLS A FUNCTION SUBROUTINE, AAWFN1, TO EVALUATE THAT PART
C
C      OF F ASSOCIATED WITH AAW THREATS.
C
C  IV. QDRTF CALLS A FUNCTION SUBROUTINE, ASWFN1, TO EVALUATE THAT PART
C
C      OF F ASSOCIATED WITH ASW THREATS. SINCE AAW THREATS ARE
C
C      INDEPENDENT OF ASW THREATS, F = - RR * SS,
C
C      WHERE,
C
C          RR = PROB(CARRIER SURVIVES GIVEN AAW THREATS), AND
C
C          SS = PROB(CARRIER SURVIVES GIVEN ASW THREATS)
C
C
C 3. SUBROUTINE AAWFN1:
C
C    IT EVALUATES THAT PART OF THE SURVIVAL PROBABILITY OF THE CARRIER
C
```

```
C    DUE TO AAW THREATS.
C
C
C 4. SUBROUTINE ASWFN1:
C
C    IT EVALUATES THAT PART OF THE SURVIVAL PROBABILTIY OF THE CARRIER
C
C    DUE TO ASW THREATS.
C
C
C 5. SUBROUTINE READ3:
C
C    IT READS IN ALL THE INPUT PARAMETERS.
C
C
C
C
C
C THE FOLLOWING ARE THE PARAMETERS THAT APPEAR IN THIS PROGRAM.
C
C THEY APPEAR IN ALPHABETICAL ORDER.
C
C
C**********************************************************************
C
C                      PARAMETER USAGE
C
C**********************************************************************
C
C
C        * * *  INPUT PARAMETERS  * * *
C
C
C AAWEXP(I).... THE NUMBER OF TARGETS IN THE ITH AAW THREAT SECTOR.
C
C AAWLL(I)..... THE LOWER LIMIT OF THE ITH AAW THREAT SECTOR.
C
C AAWUL(I)..... THE UPPER LIMIT OF THE ITH AAW THREAT SECTOR.
C
C ASIGMA(I).... THE AAW CAPABILITY FACTOR OF THE ITH SHIP.
C
C ASWEXP(I).... THE NUMBER OF TARGETS IN THE ITH ASW THREAT SECTOR.
C
C ASWLL(I)..... THE LOWER LIMIT OF THE ITH ASW THREAT SECTOR.
C
C ASWUL(I)..... THE UPPER LIMIT OF THE ITH ASW THREAT SECTOR.
```

```
C
C G............ THE NUMBER OF AAW THREAT SECTORS.
C
C H............ THE NUMBER OF ASW THREAT SECTORS.
C
C KA(I)........ AN AAW PARAMETER REFLECTING THE EFFECT OF THE ITH SHIP'S
C
C               DISTANCE TO THE CARRIER.
C
C KS(I)........ AN ASW PARAMETER REFLECTING THE EFFECT OF THE ITH SHIP'S
C
C               DISTANCE TO THE CARRIER.
C
C M............ THE TOTAL NUMBER OF SHIPS (AAW AND ASW).
C
C NN........... THE TOTAL NUMBER OF SHIP COORDINATES (TWICE THE NUMBER
C
C               OF SHIPS).
C
C SSIGMA(I).... THE ASW CAPABILITY FACTOR OF THE ITH SHIP.
C
C
C
C
C
C        * * *  INTERNAL ARRAYS REQUIRED BY SUMSNO  * * *
C
C
C WE GIVE A BRIEF DESCRIPTION OF THE ARRAYS WHICH SUMSNO USES
C
C INTERNALLY. FOR A MORE DETAILED DESCRIPTION, AND FOR MORE INFORMATION
C
C ABOUT SUMSNO AND ITS ARRAYS AND SUBROUTINES, PLEASE REFER TO THE
C
C REFERENCES OF THIS REPORT.
C
C
C D............ (INPUT/OUTPUT) A SCALE VECTOR SUCH THAT D(I)*X(I),
C
C               I = 1,2....,N ARE ALL IN THE COMPARABLE UNITS. SINCE
C
C               WE REQURE NO SCALING, D IS INITIALIZED TO ONES. THE
C
C               DEFAULTS PROVIDED BY DFALTU (SEE BELOW) REQUIRE THE
C
C               CALLER TO SUPPLY D.
```

```
C
C DFALTU(IV,V). (INPUT) A SUBROUTINE OF INPUT DEFAULT VALUES. SEE BELOW.
C
C IV........... (INPUT/OUTPUT) AN INTEGER VALUE ARRAY OF LENGTH AT
C
C               LEAST 39 THAT HELPS CONTROL THE SUMSNO ALGORITHM. IT IS
C
C               ALSO USED TO STORE SEVERAL INTERMEDIATE QUANTITIES.
C
C N............ (INPUT) THE NUMBER OF VARIABLES ON WHICH F DEPENDS,
C
C               I.E. THE NUMBER OF COMPONENTS IN X.
C
C QDRTF........ (INPUT) A SUBRROUTINE THAT, GIVEN X, CALCULATES F(X).
C
C               QDRTF MUST BE DECLARED EXTERNAL IN THE CALLING PROGRAM.
C
C               IT IS INVOKED BY
C
C                 CALL QDRTF(N, X, NF, F, UIP, URP, UFP)
C
C               NF IS THE INVOCATION COUNT FOR QDRTF. THE OTHER
C
C               PARAMETERS ARE AS DESCRIBED ABOVE AND BELOW.
C
C SUMSNO....... MINIMIZE (MAXIMIZE) GENERAL UNCONSTRAINED OBJECTIVE
C
C               FUNCTION USING FINITE-DIFFERENCE GRADIENTS AND SECANT
C
C               HESSIAN APPROXIMATIONS.
C
C X............ (INPUT/OUTPUT) BEFORE (INITIALLY) CALLING SUMSNO, THE
C
C               CALLER SHPULD SET X TO AN INITIAL GUESS AT X*. WHEN
C
C               SUMSNO RETURNS, X CONTAINS THE BEST POINT SO FAR
C
C               FOUND.
C
C UFP.......... (INPUT) USER EXTERNAL SUBROUTINE OR FUNCTION PASSED
C
C               WITHOUT CHANGE TO QDRTF.
C
C UIP.......... (INPUT) USER INTEGER PARAMETER ARRAY PASSED WITHOUT
C
C               CHANGE TO QDRTF.
```

```
C
C URP.......... (INPUT) USER FLOATING-POINT PARAMETER ARRAY PASSED
C
C               WITHOUT CHANGE TO QDRTF.
C
C V............ (INPUT/OUTPUT) A FLOATING-POINT VALUE ARRAY OF LENGTH
C
C               AT LEAST 67 + (N*(N+19)/2) THAT HELPS CONTROL THE
C
C               SUMSNO ALGORITHM. IT IS ALSO USED TO STORE SEVERAL
C
C               INTERMEDIATE QUANTITIES.
C
C
C
C
C
C                * * *  OUTPUT PARAMETERS  * * *
C
C
C ANGLE(I)..... THE OPTIMAL ANGLE OF THE ITH SHIP.
C
C RAD(I)....... THE OPTIMAL SHIP-TO-CARRIER DISTANCE OF THE ITH SHIP.
C
C
C
C
C
C                 * * *  MISCELLANEOUS  * * *
C
C
C C, E, I,..... INTEGERS USED FOR INDEXING PURPOSES.
C
C DATAN........ DOUBLE PRECISION ARCTANGENT FUNCTION.
C
C DSQRT........ DOUBLE PRECISION SQUARE ROOT FUNCTION.
C
C MINNUM....... IMPLEMENTATION DEPENDENT CONSTANT. THE SMALLEST
C
C               EXPONENT.
C
C PHI(I)....... THE OPTIMAL ANGLE OF THE ITH SHIP. IT IS USED IN THE
C
C               PROGRAM, BUT NOT PRINTED OUT.
C
C PI........... THE IRRATIONAL NUMBER, 3.141592653589793
```

```
C
C SMALL........ IMPLEMENTATION DEPENDENT CONSTANT. THE SMALLEST
C
C              NUMBER THAT CAN BE EVALUATED.
C
C Y(I)......... ITS THE VALUE OF Y(I)/X(I), WHERE, (X(I),Y(I)) ARE THE
C
C              OPTIMAL CARTESIAN COORDINATES OF THE ITH SHIP. IT IS
C
C              USED IN THE PROGRAM, BUT IS NOT PRINTED OUT.
C
C
C
C
C                * * * DECLARATION OF VARIABLES * * *
C
C
      EXTERNAL DFALTU, QDRTF, SUMSNO
      INTEGER I, IV(39), UIP(1)
      DOUBLE PRECISION D(20), URP(20,3), V(457), X(20)
      DOUBLE PRECISION KA(150),KS(150), ASIGMA(150), SSIGMA(150)
      DOUBLE PRECISION AAWEXP(150), ASWEXP(150)
      DOUBLE PRECISION PHI(150), RAD(150)
      DOUBLE PRECISION DATAN, DSQRT
      DOUBLE PRECISION Y(150), ANGLE(150)
      DOUBLE PRECISION AAWLL(150), AAWUL(150), ASWLL(150), ASWUL(150)
      REAL*8 PI
      REAL*8 MINNUM
      REAL*8 SMALL
      INTEGER C, E, G, H, M, NN
      COMMON /V/ KA
      COMMON /VV/ KS
      COMMON /U/ ASIGMA
      COMMON /W/ SSIGMA
      COMMON /C/ AAWEXP
      COMMON /D/ ASWEXP
      COMMON /Y/ AAWLL, AAWUL
      COMMON /Z/ ASWLL, ASWUL
      COMMON /ASECT/ G
      COMMON /SSECT/ H
      COMMON /SHIP/ M
      COMMON /X/ X
      COMMON /MIN/ MINNUM
      COMMON /SM/ SMALL
      PI = 3.141592653589793
C
```

```
C
C
C
C
C      * * * HOW THE VALUES ARE READ IN THE DATA FILE  * * *
C
C
C     THE VALUES ARE READ IN THE FOLLOWING ORDER:
C
C     MINNUM   SMALL
C     NN
C     D(I)
C     X(I)
C     M
C     KA(I)
C     KS(I)
C     ASIGMA(I)
C     SSIGMA(I)
C     G
C     AAWLL(I)
C     AAWUL(I)
C     AAWEXP(I)
C     H
C     ASWLL(I)
C     ASWUL(I)
C     ASWEXP(I)
C
C
C
C
C
C    * * *  CALL READ3 TO GET THE INPUT FROM THE DATA FILE  * * *
C
C
      CALL READ3(G,H,M,NN,D,X,KA,KS,ASIGMA,SSIGMA,AAWLL,AAWUL,AAWEXP,
     1ASWLL,ASWUL,ASWEXP,MINNUM,SMALL)
C
C
C
C
C
C         * * *  CONVERT SECTOR DEGREES TO RADIANS  * * *
C
C     THE LIMITS OF THE SECTORS WERE GIVEN IN DEGREES. WE NEED TO GIVE
C     THE LIMITS IN RADIANS. THE NEXT TWO "DO LOOPS" CONVERT THE DEGREES
C     TO RADIANS.
```

```
C
C
C
      DO 51 I = 1,G
         AAWLL(I) = AAWLL(I) * PI/180.D+0
         AAWUL(I) = AAWUL(I) * PI/180.D+0
 51   CONTINUE
C
      DO 52 I = 1,H
         ASWLL(I) = ASWLL(I) * PI/180.D+0
         ASWUL(I) = ASWUL(I) * PI/180.D+0
 52   CONTINUE
C
C
C
C
C
C             * * *  PRINT VARIOUS INFORMATION  * * *
C
      WRITE (6,400)
 400  FORMAT (1X, 'KA(I) IS AAW PARAMETER OF ITH SHIP REFLECTING',
     1' ','EFFECT OF ITS DISTANCE TO CARRIER')
      WRITE (6,16) (I, KA(I), I = 1,M)
 16   FORMAT (1X, 'KA(',I2,') = ', F10.5)
C
C
      WRITE (6,401)
 401  FORMAT ('0', 'KS(I) IS ASW PARAMETER OF ITH SHIP REFLECTING',
     1' ','EFFECT OF ITS DISTANCE TO CARRIER')
      WRITE (6,161) (I, KS(I), I = 1,M)
 161  FORMAT (1X, 'KS(',I2,') = ', F10.5)
C
C
      WRITE (6,402)
 402  FORMAT ('0', 'ASIGMA(I) IS THE AAW CAPABILITY FACTOR OF SHIP I')
      WRITE (6,17) (I, ASIGMA(I), I = 1,M)
 17   FORMAT (1X, 'ASIGMA(', I2,') = ', F10.5)
C
C
      WRITE (6,403)
 403  FORMAT ('0', 'SSIGMA(I) IS THE ASW CAPABILITY FACTOR OF SHIP I')
      WRITE (6,18) (I, SSIGMA(I), I = 1,M)
 18   FORMAT (1X, 'SSIGMA(', I2,') = ', F10.5)
C
C
      WRITE (6,404)
```

```
 404  FORMAT ('0', 'AAWLL(I) IS THE LOWER LIMIT OF THE ITH AAW SECTOR')
      WRITE (6,19) (I, AAWLL(I), I = 1,G)
 19   FORMAT (1X, 'AAWLL(',I2,') = ', F10.5)
C
C
      WRITE (6,405)
 405  FORMAT ('0', 'AAWUL(I) IS THE UPPER LIMIT OF THE ITH AAW SECTOR')
      WRITE (6,20) (I, AAWUL(I), I = 1,G)
 20   FORMAT (1X, 'AAWUL(',I2,') = ', F10.5)
C
C
      WRITE (6,406)
 406  FORMAT ('0', 'ASWLL(I) IS THE LOWER LIMIT OF THE ITH ASW SECTOR')
      WRITE (6,21) (I, ASWLL(I), I = 1,H)
 21   FORMAT (1X, 'ASWLL(',I2,') = ', F10.5)
C
C
      WRITE (6,407)
 407  FORMAT ('0', 'ASWUL(I) IS THE UPPER LIMIT OF THE ITH ASW SECTOR')
      WRITE (6,22) (I, ASWUL(I), I = 1,H)
 22   FORMAT (1X, 'ASWUL(',I2,') = ', F10.5)
C
C
      WRITE (6,408)
 408  FORMAT ('0', 'AAWEXP(I) IS THE NUMBER OF TARGETS IN AAW SECTOR I')
      WRITE (6,23) (I, AAWEXP(I), I = 1,G)
 23   FORMAT (1X, 'AAWEXP('I2,') = ', F10.5)
C
C
      WRITE (6,409)
 409  FORMAT ('0', 'ASWEXP(I) IS THE NUMBER OF TARGETS IN ASW SECTOR I')
      WRITE (6,24) (I, ASWEXP(I), I = 1,H)
 24   FORMAT (1X, 'ASWEXP('I2,') = ', F10.5)
C
C
      WRITE (6,25)
 25   FORMAT (16HONOPNGN ON QDRTF)
      IV(1)=0
C
C
C
C
C     WE USE THE SUMSNO SUBROUTINE WHICH MINIMIZES (MAXIMIZES) A GENERAL
C     UNCONSTRAINED OBJECTIVE FUNCTION USING A GRADIENT
C     AND A HESSIAN APPROXIMATION. THE MAIN PROGRAM REPEATEDLY
C     MINIMIZES (MAXIMIZES) THE OBJECTIVE FUNCTION.
```

```
C     THE ROUTINE QDRTF EVALUATES THE OBJECTIVE FUNCTION F(X)
C     DESCRIBED IN THE MAIN PROGRAM.
C
C
C
C        * * *  CALL SUMSNO TO MAXIMIZE THE OBJECTIVE FUNCTION  * * *
C
C
      CALL SUMSNO(NN, D, X, QDRTF, IV, V, UIP, URP, QDRTF)
C
C
C
C**********************************************************************
C
C     WE NOW COMPUTE THE ANGLES OF THE LOCATIONS OF THE SHIPS.
C     "PHI(I)" IS THE ANGLE ASSOCIATED WITH SHIP I AT ITS FINAL
C     (OPTIMAL) LOCATION.
C
C**********************************************************************
C
C     WE COMPUTE THE ANGLE AS FOLLOWS: LET SHIP I HAVE FINAL COORDINATES
C     (X(E),X(F)). THEN THE ANGLE PHI(I) ASSOCIATED WITH SHIP I'S
C     FINAL (OPTIMAL) POSITION IS GIVEN BY PHI(I) = ARCTAN(X(F)/X(E)),
C     WHICH IN THIS PROGRAM IS WRITTEN AS PHI(I) = DATAN(Y(I)). THIS
C     COMPUTATION RETURNS PHI(I) IN RADIANS. WE WOULD LIKE TO HAVE
C     PHI(I) IN DEGREES, FOR SIMPLICITY SAKE. IN ORDER TO CONVERT TO
C     DEGREES, WE NEED TO CONSIDER ALL POSSIBLE COMBINATIONS OF
C     (X(E),X(F)) - I.E. ALL QUADRANTS. THIS IS SO BECAUSE, FOR EXAMPLE,
C     ARCTAN (I.E. DATAN) MAKES NO DISTINCTION BETWEEN (X(F)/X(E)) AND
C     (-X(E)/-X(F)), X(E), X(F) > 0. SO IT DOES NOT DIFFERENTIATE
C     BETWEEN QUADRANTS 1 AND 3, AND QUADRANTS 2 AND 4.
C     THIS EXPLAINS ALL THE "IF" STATEMENTS.
C
C
      PI = 3.141592653589793
      DO 94 I = 1,M
            C = (2 * I) - 1
            E = (2 * I)
         Y(I) = X(E)/X(C)
       PHI(I) = DATAN(Y(I))
      IF(X(C) .GE. 0.D+0 .AND. X(E) .GE. 0.D+0) ANGLE(I) = (180.D+0
     1* PHI(I))/PI
      IF(X(C) .LE. 0.D+0 .AND. X(E) .GE. 0.D+0) ANGLE(I) = ((180.D+0
     1* PHI(I))/PI) + 180.D+0
      IF(X(C) .GE. 0.D+0 .AND. X(E) .LE. 0.D+0) ANGLE(I) = ((180.D+0
     2* PHI(I))/PI) + 360.D+0
```

```
      IF(X(C) .LE. 0.D+0 .AND. X(E) .LE. 0.D+0) ANGLE(I) = ((180.D+0
     3* PHI(I))/PI) + 180.D+0
      WRITE(6,90) I, ANGLE(I)
   90 FORMAT(1X, 'ANGLE(', I2, ') = ', F9.3, 3X, 'DEGREES')
  94  CONTINUE
C
C
C     WE COMPUTE THE RADIUS OF EACH OF THE M SHIPS, RAD(I).
C
      DO 103 I = 1,M
           C = (2 * I) -1
           E = (2 * I)
      RAD(I) = DSQRT ((X(C) * X(C)) + (X(E) * X(E)))
      WRITE (6,30) I, RAD(I)
  30  FORMAT (1X, 'RAD(', I2,') = ', F10.5)
 103  CONTINUE
C
      STOP
      END
C
C
C
C
C
C***********************************************************************
C
C     QDRTF
C
C***********************************************************************
C
C     SUBROUTINE QDRTF IS THE ONE THAT EVALUATES THE OBJECTIVE FUNCTION
C     F(X).
C     SINCE F(X) IS AN INTEGRAL , WE NEED A SUBROUTINE TO ESTIMATE THE
C     VALUE OF THE INTEGRAL. THIS IS DONE BY D01BAF.
C     MOST OF THE PARAMETERS WERE EXPLAINED AT THE BEGINNING OF THE
C     PROGRAM. ANY NEW PARAMETERS APPEARING HERE ARE EXPLAINED BELOW.
C
C**********************************************************************
C
C                     PARAMETER USAGE
C
C**********************************************************************
C
C
C AAWANS(I).... THE VALUE OF THE INTEGRAL DUE TO A SINGLE THREAT IN THE
C
```

```
C               ITH AAW THREAT SECTOR, IS RETURNED AS AAWANS(I).
C
C AAWFN1....... A REAL FUNCTION CALLED BY D01BAF TO EVALUATE THE
C
C               INTEGRAND.
C
C               AS A RESULT, IT IS ALSO THE INTEGRAND; I.E THE SURVIVAL
C
C               PROBABILITY OF THE CARRIER DUE TO A THREAT IN THE ITH
C
C               AAW SECTOR.
C
C AAWTOT(I).... THE SURVIVAL PROBABILITY OF THE CARRIER DUE TO ANY
C
C               NUMBER OF THREATS IN THE ITH AAW SECTOR. IT IS
C
C               DETERMINED AS:
C
C               AAWTOT(I) = AAWANS(I) ** AAWEXP(I),
C
C               WHERE, AAWEXP(I) IS THE NUMBER OF TARGETS IN THE ITH
C
C               AAW SECTOR.
C
C ASWANS(I).... THE VALUE OF THE INTEGRAL DUE TO A SINGLE THREAT IN THE
C
C               ITH ASW SECTOR, IS RETURNED AS ASWANS(I).
C
C ASWFN1....... A REAL FUNCTION CALLED BY D01BAF TO EVALUATE THE
C
C               INTEGRAND.
C
C               AS A RESULT, IT IS ALSO THE INTEGRAND; I.E. THE
C
C               SURVIVAL PROBABILITY OF THE CARRIER DUE TO A THREAT IN
C
C               THE ITH ASW SECTOR.
C
C ASWTOT(I).... THE SURVIVAL PROBABILTIY OF THE CARRIER DUE TO ANY
C
C               NUMBER OF THREATS IN THE ITH ASW SECTOR. THE VALUE
C
C               IS DETERMINED AS:
C
C               ASWTOT(I) = ASWANS(I) ** ASWEXP(I),
C
```

```
C               WHERE, ASWEXP(I) IS THE NUMBER OF TARGETS IN THE ITH
C
C               ASW SECTOR.
C
C DEXP......... DOUBLE PRECISION EXPONENTIAL FUNCTION.
C
C DO1BAF....... A FUNCTION THAT COMPUTES AN ESTIMATE OF THE DEFINITE
C
C               INTEGRALS, AAWFN1 AND ASWFN1.
C
C DO1BAZ....... A SUBROUTINE PROVIDED BY THE NAG LIBRARY USED TO
C
C               EVALUATE AN INTEGRAL ON A FINITE INTERVAL.
C
C IFAIL........ THIS IS AN ERROR INDICATOR.
C
C II........... USED FOR INDEXING PURPOSES.
C
C NF........... THE NUMBER OF FUNCTION EVALUATIONS.
C
C NSTOR(1)..... THE NUMBER OF ABSCISSAE USED IN THE EVALUATION OF THE
C
C               INTEGRAL.
C
C RR........... VALUE OF THE SURVIVAL PROBABILITY OF THE CARRIER DUE
C
C               TO AAW THREATS FROM ALL THE AAW SECTORS, WHEN THERE ARE
C
C               ONLY AAW TYPE THREATS.
C
C SS........... VALUE OF THE SURVIVAL PROBABILITY OF THE CARRIER DUE
C
C               TO ASW THREATS FROM ALL THE ASW SECTORS, WHEN THERE ARE
C
C               ONLY ASW TYPE THREATS.
C
C T1........... VALUE OF THE SURVIVAL PROBABILITY OF THE CARRIER DUE
C
C               TO AAW THREATS FROM ALL THE AAW SECTORS, WHEN THERE ARE
C
C               BOTH AAW AND ASW TYPE THREATS.
C
C T2........... VALUE OF THE SURVIVAL PROBABILITY OF THE CARRIER DUE
C
C               TO ASW THREATS FROM ALL THE ASW SECTORS, WHEN THERE ARE
C
```

```
C                BOTH AAW AND ASW TYPE THREATS.
C
C
C
C
C
      SUBROUTINE QDRTF(N, X, NF, F, UIP, URP, UFP)
      INTEGER N,NF,UIP(1)
      DOUBLE PRECISION X(N), F, URP(N,3)
      EXTERNAL UFP
      DOUBLE PRECISION DEXP, DSQRT
      DOUBLE PRECISION KA(150),KS(150), ASIGMA(150), SSIGMA(150)
      REAL*8 PI
      DOUBLE PRECISION AAWEXP(150), AAWTOT(150), AAWANS(150)
      DOUBLE PRECISION ASWEXP(150), ASWTOT(150), ASWANS(150)
      REAL*8 RR, SS, T1, T2
      DOUBLE PRECISION AAWLL(150), AAWUL(150)
      DOUBLE PRECISION ASWLL(150), ASWUL(150)
      INTEGER I, IFAIL, II
      INTEGER C, E, G, H, M
      INTEGER NSTOR(1)
      REAL*8 D01BAF
      DOUBLE PRECISION AAWFN1, ASWFN1
      EXTERNAL D01BAZ, AAWFN1, ASWFN1
      COMMON /INDEX/ II
      COMMON /V/ KA
      COMMON /VV/ KS
      COMMON /U/ ASIGMA
      COMMON /W/ SSIGMA
      COMMON /C/ AAWEXP
      COMMON /D/ ASWEXP
      COMMON /Y/ AAWLL, AAWUL
      COMMON /Z/ ASWLL, ASWUL
      COMMON /ASECT/ G
      COMMON /SSECT/ H
      COMMON /SHIP/ M
      UIP(1) = NF
      NSTOR(1)=16
C
C
C
C
C
      DO 250 I = 1, N
         URP(I,2) = URP(I,1)*X(I)
 250  CONTINUE
```

```
C
C
C
C
C
C      * * *  DETERMINING THE THREAT SECTORS  * * *
C
C     G = 0 : NO AAW SECTORS
C     H = 0 : NO ASW SECTORS
C
C**********************************************************************
C
C     IF THERE ARE NO AAW SECTORS (I.E. NO AAW THREATS), THEN G = 0
C
C     AND WE GO TO 1111 TO EVALUATE FOR ASW THREATS.
C
C
      IF(G .EQ. 0) GO TO 1111
C
C**********************************************************************
C
C**********************************************************************
C
C     IF THERE ARE NO ASW SECTORS (I.E. NO ASW THREATS), H = 0 AND
C
C     WE GO TO 1112 TO EVALUATE FOR AAW THREATS.
C
C
      IF(H .EQ. 0) GO TO 1112
C
C**********************************************************************
C**********************************************************************
C
C     IF THERE ARE BOTH AAW AND ASW THREATS, THEN GO TO 1113
C
C
      IF((G .GT. 0) .AND. (H .GT. 0)) GO TO 1113
C
C**********************************************************************
C     GO TO 1113
C
C
C
C
C
C  * * *  CALL INTEGRATION ROUTINE REPEATEDLY OVER G AAW SECTORS  * * *
```

```
C
C
 1112 DO 65 II = 1,G
      IFAIL=1
      AAWANS(II) = D01BAF(D01BAZ,AAWLL(II),AAWUL(II),NSTOR(1),
     1             AAWFN1,IFAIL)
  65  CONTINUE
C
C
C
C
C
C   * * CALCULATE TOTAL SURVIVAL PROBABILITY DUE TO ALL AAW SECTORS * *
C
C
      RR = 1.D+0
      DO 39 I = 1, G
         AAWTOT(I) = AAWANS(I) ** AAWEXP(I)
                RR = RR * AAWTOT(I)
  39  CONTINUE
      F = -RR
      GO TO 999
C
C
C
C
C
C  * * *  CALL INTEGRATION ROUTINE REPEATEDLY OVER H ASW SECTORS  * * *
C
 1111 DO 66 II = 1,H
      IFAIL = 1
      ASWANS(II) = D01BAF(D01BAZ,ASWLL(II),ASWUL(II),NSTOR(1),
     2             ASWFN1,IFAIL)
  66  CONTINUE
C
C
C
C
C
C * * CALCULATE TOTAL SURVIVAL PROBABILITY DUE TO ALL ASW SECTORS * *
C
C
      SS = 1.0D+0
      DO 38 I = 1,H
      ASWTOT(I) = ASWANS(I) ** ASWEXP(I)
             SS = SS * ASWTOT(I)
```

```
   38 CONTINUE
      F = -SS
      GO TO 999
C
C
C
C
C
C * * CALCULATE SURVIVAL PROBABILITY DUE TO AAW AND ASW SECTORS * *
C
C
 1113 DO 67 II = 1,G
      IFAIL = 1
      AAWANS(II) = D01BAF(D01BAZ,AAWLL(II),AAWUL(II),NSTOR(1),
     1                    AAWFN1,IFAIL)
   67 CONTINUE
C
C
      T1 = 1.D+0
      DO 34 I = 1,G
         AAWTOT(I) = AAWANS(I) ** AAWEXP(I)
                T1 = T1 * AAWTOT(I)
   34 CONTINUE
C
C
      DO 68 II = 1,H
      IFAIL = 1
      ASWANS(II) = D01BAF(D01BAZ,ASWLL(II),ASWUL(II),NSTOR(1),
     2                    ASWFN1,IFAIL)
   68 CONTINUE
C
C
C
      T2 = 1.D+0
      DO 35 I = 1,H
         ASWTOT(I) = ASWANS(I) ** ASWEXP(I)
                T2 = T2 * ASWTOT(I)
   35 CONTINUE
C
C             * * * CALCULATE OBJECTIVE FUNCTION * * *
C
      F = -(T1 * T2)
      GO TO 999
  999 RETURN
      END
C
```

```
C
C
C
C
C*********************************************************************
C
C     AAWFN1
C
C*********************************************************************
C
C
C
C THE FUNCTION SUBPROGRAM AAWFN1 CALCULATES THE SURVIVAL PROBABILITY
C
C OF THE CARRIER IN A GIVEN AAW THREAT SECTOR. WE FORMULATE THE
C
C INTEGRAND, AAWFN1, WHICH IS USED BY D01BAF.
C
C
C
C********************************************************************
C
C                     PARAMETER USAGE
C
C *******************************************************************
C
C AAWT......... = Π( 1 - PSHIP(I) ) = THE PROBABILTIY THAT NO SHIP
C
C                  KILLS THE AAW TARGET.
C
C KLUGE(I)..... THE OPTIMUM INTERCEPT DISTANCE OF SHIP I TO A TARGET.
C
C PR(I)........ THE EFFECT OF THE ITH SHIP'S DISTANCE FROM THE CARRIER.
C
C PSHIP(I)..... THE PROBABILITY THAT SHIP I KILLS A TARGET.
C
C R(I)......... THE DISTANCE OF THE ITH SHIP FROM THE CARRIER, I.E.
C
C               FROM THE ORIGIN.
C
C THETA........ THE ANGLE OF APPROACH OF AN AAW TARGET. IT IS
C
C               UNIFORMLY DISTRIBUTED OVER A GIVEN AAW SECTOR.
C
C********************************************************************
C
```

```
C
C
C
C
      FUNCTION AAWFN1(THETA)
      DOUBLE PRECISION X(20)
      REAL*8 THETA
      REAL*8 MINNUM
      REAL*8 SMALL
      REAL*8 AAWT
      INTEGER C, E, I, II, M
      DOUBLE PRECISION KLUGE(150), PSHIP(150)
      DOUBLE PRECISION AAWLL(150), AAWUL(150)
      DOUBLE PRECISION KA(150), ASIGMA(150)
      DOUBLE PRECISION R(150), PR(150)
      DOUBLE PRECISION DSQRT, DEXP
      COMMON /INDEX/ II
      COMMON /V/ KA
      COMMON /U/ ASIGMA
      COMMON /SHIP/ M
      COMMON /Y/ AAWLL, AAWUL
      COMMON /X/ X
      COMMON /MIN/ MINNUM
      COMMON /SM/ SMALL
C
C       * * *  COMPUTE CARRIER SURVIVAL PROBABILITY  * * *
C
      AAWT = 1.D+0
      DO 79 I = 1,M
            C = (2 * I) - 1
            E = (2 * I)
                R(I) = DSQRT((X(C)*X(C)) + (X(E)*X(E)))
               PR(I) = 1.D+0 - DEXP(-KA(I) * R(I))
            KLUGE(I) = (X(C)*DSIN(THETA)) - (X(E)*DCOS(THETA))
      IF ((-(KLUGE(I)*KLUGE(I)/ASIGMA(I)) .LE. MINNUM)
     1.OR. (PR(I) * DEXP(-(KLUGE(I) * KLUGE(I)/ASIGMA(I)))
     2.LE. SMALL)) PSHIP(I) = 0.D+0
         PSHIP(I) = PR(I) * DEXP(-(KLUGE(I) * KLUGE(I))/ASIGMA(I))
             AAWT = AAWT * (1.D+0 -PSHIP(I))
   79 CONTINUE
C
C              * * *  THE INTEGRAND  * * *
C
      AAWFN1= ((1.D+0)/(AAWUL(II)-AAWLL(II))) * (1.D+0 - AAWT)
C
C
```

```
      RETURN
      END
C
C
C
C
C
C*********************************************************************
C
C     ASWFN1
C
C*********************************************************************
C
C
C
C THE FUNCTION SUBPROGRAM AAWFN1 IS USED TO CALCULATE THE SURVIVAL
C
C PROBABILITY OF THE CARRIER IN A GIVEN ASW THREAT SECTOR. WE FORMULATE
C
C THE INTEGRAND, ASWFN1, WHICH IS USED BY DO1BAF.
C
C
C
C*********************************************************************
C
C                 PARAMETER USAGE
C
C*********************************************************************
C                  M
C ASWT......... =Π( 1 - PSHIP(I) ) = THE PROBABILITY THAT NO SHIP
C               I=1
C                 KILLS THE ASW TARGET.
C
C KLUGE(I)..... THE OPTIMUM INTERCEPT DISTANCE OF SHIP I TO A TARGET.
C
C PR(I)........ THE EFFECT OF THE ITH SHIP'S DISTANCE TO THE CARRIER.
C
C PSHIP(I)..... THE PROBABILITY THAT SHIP I KILLS AN ASW TARGET.
C
C R(I)......... THE DISTANCE OF THE ITH SHIP FROM THE CARRIER, I.E.
C
C               FROM THE ORIGIN.
C
C THETA........ THE ANGLE OF APPROACH OF AN ASW TARGET. IT IS UNIFORMLY
C
C               DISTRIBUTED OVER A GIVEN ASW SECTOR.
```

```
C
C
      FUNCTION ASWFN1(THETA)
      DOUBLE PRECISION X(20)
      REAL*8 THETA
      REAL*8 MINNUM
      REAL*8 SMALL
      REAL*8 ASWT
      INTEGER C, E, I, II, M
      DOUBLE PRECISION KLUGE(150), PSHIP(150)
      DOUBLE PRECISION ASWLL(150), ASWUL(150)
      DOUBLE PRECISION KS(150), SSIGMA(150), R(150), PR(150)
      DOUBLE PRECISION DSQRT, DEXP
      COMMON /INDEX/ II
      COMMON /VV/ KS
      COMMON /W/ SSIGMA
      COMMON /SHIP/ M
      COMMON /Z/ ASWLL, ASWUL
      COMMON /X/ X
      COMMON /MIN/ MINNUM
      COMMON /SM/ SMALL
C
C
C       * * *  COMPUTE CARRIER SURVIVAL PROBABILITY   * * *
C
      ASWT = 1.D+0
      DO 81 I = 1,M
            C = (2 * I) - 1
            E = (2 * I)
             R(I) = DSQRT((X(C)*X(C)) + (X(E)*X(E)))
            PR(I) = 1.D+0 - DEXP(-KS(I) * R(I))
            KLUGE(I) = (X(C)*DSIN(THETA)) - (X(E)*DCOS(THETA))
      IF ((-(KLUGE(I)*KLUGE(I)/SSIGMA(I)) .LE. MINNUM)
     1.OR. (PR(I) * DEXP(-(KLUGE(I) * KLUGE(I)/SSIGMA(I)))
     2.LE. SMALL)) PSHIP(I) = 0.D+0
         PSHIP(I) = PR(I) * DEXP(-(KLUGE(I) * KLUGE(I))/SSIGMA(I))
             ASWT = ASWT * (1.D+0 -PSHIP(I))
   81 CONTINUE
C
C
C
C            * * *  THE INTEGRAND  * * *
C
      ASWFN1= ((1.D+0)/(ASWUL(II)-ASWLL(II))) * (1.D+0 - ASWT)
C
C
```

```
      RETURN
      END
C
C
C
C
C
C     *** SUBROUTINE READ3 READS THE VALUES FROM THE DATA FILE ***
C
C**********************************************************************
C
C      READ3
C
C**********************************************************************
C
C
C
C
C
      SUBROUTINE READ3(G,H,M,NN,D,X,KA,KS,ASIGMA,SSIGMA,AAWLL,AAWUL,
     1AAWEXP,ASWLL,ASWUL,ASWEXP,MINNUM,SMALL)
      INTEGER G,H,M,NN
      REAL*8 MINNUM,SMALL
      DOUBLE PRECISION D(20), X(20), ASIGMA(150), SSIGMA(150)
      DOUBLE PRECISION AAWLL(150), AAWUL(150), ASWLL(150), ASWUL(150)
      DOUBLE PRECISION AAWEXP(150), ASWEXP(150)
      DOUBLE PRECISION KA(150), KS(150)
C
C
C
C
C
C     * * *  READ THE IMPLEMENTATION DEPENDENT CONSTANTS  * * *
C
C
      READ(82,1) MINNUM, SMALL
    1 FORMAT(/D13.6,2X,E7.1)
C
C
C         * * * READ THE TOTAL NUMBER OF COORDINATES  * * *
C
C
      READ(82,2) NN
    2 FORMAT(///I2)
C
C
```

```
C          * * * READ THE SCALE ARRAY  * * *
C
C
      DO 10 I = 1,NN
         READ(82,3) D(I)
    3 FORMAT(D9.3)
   10 CONTINUE
C
C
C         * * *  READ THE INITIAL CONDITIONS  * * *
C
C
      DO 11 I = 1,NN
         READ(82,4) X(I)
    4 FORMAT(D9.3)
   11 CONTINUE
C
C
C              * * *  READ THE TOTAL NUMBER OF SHIPS  * * *
C
C
      READ(82,5) M
    5 FORMAT(///I2)
C
C
C         * * *  READ THE AAW DISTANCE SCALE FACTORS  * * *
C
C
      DO 12 I = 1,M
         READ(62,6) KA(I)
    6 FORMAT(D9.3)
   12 CONTINUE
C
C
C         * * *  READ THE ASW DISTANCE SCALE FACTORS  * * *
C
C
      DO 121 I = 1,M
         READ(82,61) KS(I)
   61 FORMAT(D9.3)
  121 CONTINUE
C
C
C         * * *  READ THE AAW RANGE FACTORS  * * *
C
C
```

```
      DO 13 I = 1,M
         READ(82,7) ASIGMA(I)
    7 FORMAT(D9.3)
   13 CONTINUE
C
C
C          * * *  READ THE ASW RANGE FACTORS  * * *
C
C
      DO 14 I = 1,M
         READ(82,8) SSIGMA(I)
    8 FORMAT(D9.3)
   14 CONTINUE
C
C
C          * * * READ THE TOTAL NUMBER OF AAW THREAT SECTORS  * * *
C
C
      READ(82,9) G
    9 FORMAT(///I2)
C
C
C
C
      IF(G .EQ. 0) GO TO 555
C
C
C        * * *  READ THE LOWER LIMITS OF ALL THE AAW SECTORS  * * *
C
C
      DO 15 I = 1,G
         READ(82,100) AAWLL(I)
  100 FORMAT(D9.3)
   15 CONTINUE
C
C
C        * * *  READ THE UPPER LIMITS OF ALL THE AAW SECTORS  * * *
C
C
      DO 16 I = 1,G
         READ(82,101) AAWUL(I)
  101 FORMAT(D9.3)
   16 CONTINUE
C
C
C   * * *  READ THE NUMBER OF TARGETS IN EACH AAW SECTOR  * * *
```

```
C
C
      DO 17 I = 1,G
         READ(82,102) AAWEXP(I)
  102 FORMAT(D9.3)
   17 CONTINUE
C
C
C        * * *  READ THE TOTAL NUMBER OF ASW THREAT SECTORS  * * *
C
C
  555 READ(82,103) H
  103 FORMAT(///I2)
C
C
C
C
      IF(H .EQ. 0) GO TO 999
C
C
C        * * * READ THE LOWER LIMITS OF ALL THE ASW SECTORS  * * *
C
C
      DO 18 I = 1,H
         READ(82,104) ASWLL(I)
  104 FORMAT(D9.3)
   18 CONTINUE
C
C
C        * * *  READ THE UPPER LIMITS OF ALL THE ASW SECTORS  * * *
C
C
      DO 19 I = 1,H
         READ(82,105) ASWUL(I)
  105 FORMAT(D9.3)
   19 CONTINUE
C
C
C        * * *  READ THE NUMBER OF TARGETS IN EACH ASW SECTOR  * * *
C
C
      DO 20 I = 1,H
         READ(82,106) ASWEXP(I)
  106 FORMAT(D9.3)
   20 CONTINUE
C
```

```
C
C
  999 RETURN
      END
```

APPENDIX D SAMPLE OUTPUT

D.1 SAMPLE OUTPUT

Sample output of a specific experiment follows. Included is the associated data file, KSH DT.

```
MINNUM          SMALL

-0.174673D+03  1.0E-75

C

* * *  # OF COORDINATES :  NN  * * *

C

 4

0.100D+01           = D(1)

0.100D+01           = D(2)

0.100D+01           = D(3)

0.100D+01           = D(4)

0.170D+02           = X(1)

0.170D+01           = X(2)

0.100D+02           = X(3)

0.380D+02           = X(4)

C

* * *  TOTAL # OF SHIPS  :  M  * * *

C

 2

0.200D+00           = KA(1)

0.200D+00           = KA(2)

0.100D+00           = KS(1)

0.100D+00           = KS(2)

0.100D+04           = ASIGMA(1)
```

```
0.850D+03          = ASIGMA(2)

0.500D+03          = SSIGMA(1)

0.800D+04          = SSIGMA(2)

C

* * *  # OF AAW SECTORS  : G  * * *

C

 1                          # OF AAW SECTORS

0.000D+00          = AAWLL

0.600D+02          = AAWUL

0.400D+01          = AAWEXP

C

* * *  # OF ASW SECTORS  :  H  * * *

C

 1                          # OF ASW SECTORS

0.300D+02          = ASWLL

0.900D+02          = ASWUL

0.100D+01          = ASWEXP
```

```
R; T=0.01/0.01 19:20:46
dfkg3
EXECUTION BEGINS
KA(I) IS AAW PARAMETER OF ITH SHIP REFLECTING EFFECT OF ITS DISTANCE TO CARRIER
KA( 1) =     0.20000
KA( 2) =     0.20000

KS(I) IS ASW PARAMETER OF ITH SHIP REFLECTING EFFECT OF ITS DISTANCE TO CARRIER
KS( 1) =     0.10000
KS( 2) =     0.10000

ASIGMA(I) IS THE AAW CAPABILITY FACTOR OF SHIP I
ASIGMA( 1) = 1000.00000
ASIGMA( 2) =  850.00000

SSIGMA(I) IS THE ASW CAPABILITY FACTOR OF SHIP I
SSIGMA( 1) =  500.00000
SSIGMA( 2) = 8000.00000

AAWLL(I) IS THE LOWER LIMIT OF THE ITH AAW SECTOR
AAWLL( 1) =     0.00000

AAWUL(I) IS THE UPPER LIMIT OF THE ITH AAW SECTOR
AAWUL( 1) =     1.04720

ASWLL(I) IS THE LOWER LIMIT OF THE ITH ASW SECTOR
ASWLL( 1) =     0.52360

ASWUL(I) IS THE UPPER LIMIT OF THE ITH ASW SECTOR
ASWUL( 1) =     1.57080

AAWEXP(I) IS THE NUMBER OF TARGETS IN AAW SECTOR I
AAWEXP( 1) =     4.00000

ASWEXP(I) IS THE NUMBER OF TARGETS IN ASW SECTOR I
ASWEXP( 1) =     1.00000

NOPNGN ON QORTF

    I     INITIAL X(I)      D(I)

    1     0.170000D+02     0.100D+01
    2     0.170000D+01     0.100D+01
    3     0.100000D+02     0.100D+01
    4     0.380000D+02     0.100D+01

   IT    NF      F          RELDF      PRELDF      RELDX      STPPAR      D*STEP     NPRELDF

    0     1   0.837D+00
    1     2   0.837D+00  0.374D-03  0.187D-03  0.203D-03  0.000D+00  0.177D-01  0.187D-03
    2     4   0.840D+00  0.368D-02  0.186D-02  0.203D-02  0.000D+00  0.177D+00  0.186D-02
    3     6   0.867D+00  0.313D-01  0.173D-01  0.201D-01  0.000D+00  0.173D+01  0.173D-01
    4     7   0.909D+00  0.461D-01  0.531D-01  0.835D-01  0.532D-02  0.693D+01  0.531D-01
    5     8   0.910D+00  0.844D-03  0.781D-03  0.972D-02  0.000D+00  0.781D+00  0.781D-03
    6     9   0.910D+00  0.445D-04  0.246D-04  0.778D-03  0.000D+00  0.599D-01  0.246D-04
    7    11   0.910D+00  0.297D-03  0.191D-03  0.332D-02  0.000D+00  0.245D+00  0.191D-03
    8    12   0.911D+00  0.606D-03  0.370D-03  0.416D-02  0.000D+00  0.300D+00  0.370D-03
    9    13   0.912D+00  0.169D-02  0.106D-02  0.728D-02  0.000D+00  0.577D+00  0.106D-02
   10    14   0.916D+00  0.383D-02  0.240D-02  0.102D-01  0.000D+00  0.114D+01  0.240D-02
   11    15   0.923D+00  0.800D-02  0.507D-02  0.212D-01  0.000D+00  0.251D+01  0.507D-02
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12	16	-0.936D+00	0.139D-01	0.884D-02	0.471D-01	0.000D+00	0.501D+01	0.884D-02
13	17	-0.945D+00	0.982D-02	0.103D-01	0.841D-01	0.000D+00	0.769D+01	0.103D-01
14	18	-0.947D+00	0.196D-02	0.145D-02	0.199D-01	0.000D+00	0.186D+01	0.145D-02
15	19	-0.949D+00	0.161D-02	0.126D-02	0.199D-01	0.000D+00	0.119D+01	0.126D-02
16	20	-0.949D+00	0.309D-03	0.258D-03	0.780D-02	0.000D+00	0.598D+00	0.258D-03
17	21	-0.949D+00	0.300D-04	0.270D-04	0.340D-02	0.000D+00	0.292D+00	0.270D-04
18	22	-0.949D+00	0.487D-05	0.262D-05	0.422D-03	0.000D+00	0.351D-01	0.262D-05
19	24	-0.949D+00	0.464D-04	0.302D-04	0.259D-02	0.000D+00	0.209D+00	0.302D-04
20	25	-0.949D+00	0.887D-04	0.538D-04	0.298D-02	0.000D+00	0.228D+00	0.538D-04
21	26	-0.949D+00	0.262D-03	0.164D-03	0.562D-02	0.000D+00	0.403D+00	0.164D-03
22	27	-0.950D+00	0.634D-03	0.392D-03	0.875D-02	0.000D+00	0.660D+00	0.392D-03
23	28	-0.951D+00	0.155D-02	0.971D-03	0.226D-01	0.000D+00	0.146D+01	0.971D-03
24	29	-0.954D+00	0.256D-02	0.184D-02	0.574D-01	0.000D+00	0.308D+01	0.184D-02
25	30	-0.955D+00	0.804D-03	0.781D-03	0.375D-01	0.000D 00	0.196D+01	0.781D-03
26	31	-0.955D+00	0.321D-03	0.200D-03	0.760D-02	0.000D+00	0.607D+00	0.200D-03
27	32	-0.955D+00	0.165D-03	0.192D-03	0.127D-01	0.000D+00	0.919D+00	0.192D-03
28	33	-0.955D+00	0.626D-05	0.652D-05	0.149D-02	0.000D+00	0.134D+00	0.652D-05
29	34	-0.955D+00	0.110D-06	0.867D-07	0.222D-03	0.000D+00	0.168D-01	0.867D-07
30	35	-0.955D+00	0.465D-07	0.280D-07	0.850D-04	0.000D+00	0.588D-02	0.280D-07
31	36	-0.955D+00	0.145D-06	0.900D-07	0.170D-03	0.000D+00	0.114D-01	0.900D-07
32	37	-0.955D+00	0.359D-06	0.221D-06	0.278D-03	0.000D+00	0.174D-01	0.221D-06
33	38	-0.955D+00	0.951D-06	0.589D-06	0.501D-03	0.000D+00	0.291D-01	0.589D-06
34	39	-0.955D+00	0.242D-05	0.150D-05	0.900D-03	0.000D+00	0.487D-01	0.150D-05
35	40	-0.955D+00	0.601D-05	0.375D-05	0.165D-02	0.000D+00	0.880D-01	0.375D-05
36	41	-0.955D+00	0.136D-04	0.867D-05	0.291D-02	0.000D+00	0.168D+00	0.867D-05
37	42	-0.955D+00	0.251D-04	0.166D-04	0.431D-02	0.000D+00	0.303D+00	0.166D-04
38	43	-0.955D+00	0.297D-04	0.211D-04	0.667D-02	0.000D+00	0.396D+00	0.211D-04
39	44	-0.955D+00	0.173D-04	0.134D-04	0.529D-02	0.000D+00	0.274D+00	0.134D-04
40	45	-0.955D+00	0.393D-05	0.340D-05	0.163D-02	0.000D+00	0.105D+00	0.340D-05
41	46	-0.955D+00	0.219D-06	0.202D-06	0.657D-03	0.000D+00	0.369D-01	0.202D-06
42	47	-0.955D+00	0.339D-08	0.321D-08	0.996D-04	0.000D+00	0.613D-02	0.321D-08
43	48	-0.955D+00	0.195D-10	0.190D-10	0.770D-05	0.000D+00	0.411D-03	0.190D-10

***** RELATIVE FUNCTION CONVERGENCE *****

FUNCTION	-0.955245D+00	RELDX	0.7701D-05
FUNC. EVALS	48	GRAD. EVALS	348
PRELDF	0.1900D-10	NPRELDF	0.1900D-10

I	FINAL X(I)	D(I)	G(I)
1	0.205492D+02	0.100D+01	-0.126D-08
2	0.590991D+01	0.100D+01	0.170D-08
3	0.221588D+02	0.100D+01	0.426D-08
4	0.246177D+02	0.100D+01	-0.212D-08

```
ANGLE( 1) =     16.045    DEGREES
ANGLE( 2) =     48.009    DEGREES
RAD( 1) =    21.38213
RAD( 2) =    33.12165
R; T=7 11/7.90 19.21.14
cp spool console stop close
```